a) $4\cos^2 t - 4\cos t - 3 = 0$

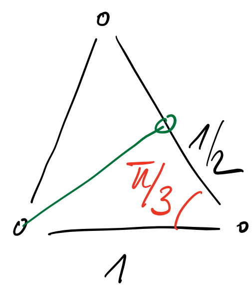
$$\boxed{\cos t = x}$$

$$4x^2 - 4x - 3 = 0$$

$$x = \frac{4 \pm \sqrt{16+48}}{8} = \begin{cases} \frac{12}{8} = \cancel{\frac{3}{2}} > 1 \\ -\frac{4}{8} = -\frac{1}{2} \end{cases}$$

$$\boxed{x = \cos t}$$

$$\cos t = -\frac{1}{2}$$



$$t = \pm \arccos\left(-\frac{1}{2}\right) + k \cdot 2\pi$$

$$t = \pm \frac{\pi}{3} + k \cdot 2\pi$$

b) $2t^2 - 3t + 1 = 0$

$$t = \frac{3 \pm \sqrt{9-8}}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

$\sin x = 1$ $\qquad x = \arcsin(1) + k \cdot 2\pi$

$x = \pi - \arcsin(1) + k \cdot 2\pi$

$\sin x = \frac{1}{2}$ $\qquad x = \arcsin\left(\frac{1}{2}\right) + k \cdot 2\pi$

$x = \pi - \arcsin\left(\frac{1}{2}\right) + k \cdot 2\pi$

c) $3 \sin^2 z + 8 \cos z + 1 = 0 \quad (*)$

On a $\sin^2 z + \cos^2 z = 1 \Leftrightarrow \sin^2 z = 1 - \cos^2 z$

On peut réécrire $(*)$:

$$3(1 - \cos^2 z) + 8 \cos z + 1 = 0$$

$$\Leftrightarrow 3 - 3\cos^2 z + 8 \cos z + 1 = 0$$

$$\Leftrightarrow 3\cos^2 z - 8 \cos z - 4 = 0$$

$$\Leftrightarrow 3t^2 - 8t - 4 = 0$$

$$t = \frac{8 \pm \sqrt{64+48}}{6} = \frac{8 \pm \sqrt{112}}{6}$$

$$\simeq \begin{cases} \cancel{3{,}97} > 1 \\ -0{,}4305 \end{cases}$$

$$\cos z = -0{,}4305$$

$$z = \pm \arccos(-0{,}4305) + k \cdot 2\pi$$

d) $3 \sin^2 t + \cos^2 t - 2 = 0$

$$\sin^2 t = 1 - \cos^2 t$$

$$\Leftrightarrow 3(1 - \cos^2 t) + \cos^2 t - 2 = 0$$

$$\Leftrightarrow 3 - 3\cos^2 t + \cos^2 t - 2 = 0$$

$$\Leftrightarrow 1 - 2\cos^2 t = 0$$

$$\Leftrightarrow \cos^2 t = \frac{1}{2} \quad \Leftrightarrow \quad \cos t = \pm \frac{1}{\sqrt{2}}$$

$$t = \pm \arccos\left(\frac{1}{\sqrt{2}}\right) + k \cdot 2\pi$$

$$= \pm \frac{\pi}{4} + k \cdot 2\pi \ .$$

En effet, $\arccos\left(\pm \frac{1}{\sqrt{2}}\right) = \pm \frac{\pi}{4}$

e) $5 \sin x = 6 \cos^2 x$

$5 \sin x = 6(1 - \sin^2 x)$

$5 \sin x = 6 - 6 \sin^2 x$

$6 \sin^2 x + 5 \sin x - 6 = 0$

$6t^2 + 5t - 6 = 0$

$$t = \frac{-5 \pm \sqrt{25 + 4 \cdot 36}}{12} = \frac{-5 \pm 13}{12} \begin{cases} 2/3 \\ \cancel{-3/2} \end{cases}$$

$$x = \arcsin(2/3) + k \cdot 2\pi$$

$$x = \pi - \arcsin(2/3) + k \cdot 2\pi$$

f) $\cos x = \dfrac{\sin x}{\cos x}$ $\qquad x \neq \dfrac{\pi}{2} + k\pi$

$$\Leftrightarrow \cos^2 x = \sin x$$

$$\Leftrightarrow 1 - \sin^2 x = \sin x$$

$$\Leftrightarrow \sin^2 x + \sin x - 1 = 0$$

$$t^2 + t - 1 = 0$$

$$t = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \begin{cases} 0{,}618 \\ \cancel{-1{,}618} \end{cases}$$

$$x = \arcsin(0{,}618) + k \cdot 2\pi$$

$$x = \pi - \arcsin(0{,}618) + k \cdot 2\pi$$

g) $8\cos^2 t + 5\sin t - 1 = 0$

$\Leftrightarrow 8(1-\sin^2 t) + 5\sin t - 1 = 0$

$\Leftrightarrow 8 - 8\sin^2 t + 5\sin t - 1 = 0$

$\Leftrightarrow 8\sin^2 t - 5\sin t - 7 = 0$

$\Leftrightarrow 8x^2 - 5x - 7 = 0$

$\Leftrightarrow x = \dfrac{5 \pm \sqrt{25 + 224}}{16} = \dfrac{5 \pm \sqrt{249}}{16}$

$x \approx \begin{cases} \cancel{1{,}2987} > 1 \\ -0{,}6737 \end{cases}$

$t = \arcsin(-0{,}6737) + k \cdot 2\pi$

$t = \pi - \arcsin(-0{,}6737) + k \cdot 2\pi$

h) $\tan^4 t - 4\tan^2 t + 3 = 0$

$\Leftrightarrow x^4 - 4x^2 + 3 = 0$

$\Leftrightarrow (x^2-1)(x^2-3) = 0$

$\Leftrightarrow (x+1)(x-1)(x+\sqrt{3})(x-\sqrt{3}) = 0$

Et donc, $\tan t \in \{\pm 1; \pm\sqrt{3}\}$

$$t \in \left\{ \pm\frac{\pi}{4} + k\cdot\pi \,;\, \pm \arctan(\sqrt{3}) + k\cdot\pi \right\}$$