a) $\sin t = 3 \cdot \cos t$

On peut supposer que $\cos t \neq 0$ $\left(t \neq \pm \frac{\pi}{2} + k 2\pi\right)$

$$\frac{\sin t}{\cos t} = 3$$

$$\tan t = 3 \iff t = \arctan 3 + k \cdot \pi$$

b) $\sin \alpha \cos \alpha - 2 \sin^2 \alpha = 0$

$$\alpha = \pm \frac{\pi}{2} + k \cdot 2\pi$$

$$\underbrace{\sin\left(\pm \frac{\pi}{2}\right) \cos\left(\pm \frac{\pi}{2}\right)}_{0} - 2 \sin^2\left(\pm \frac{\pi}{2}\right) = 0$$

$$0 \quad - 2 \cdot 1 = 0$$

$\Rightarrow$ α n'est pas solution dans ce cas.

On peut donc supposer $\alpha \neq \pm \frac{\pi}{2} + k \cdot 2\pi$

$$\frac{\sin\alpha\cos\alpha}{\cos^2\alpha} - 2\cdot\frac{\sin^2\alpha}{\cos^2\alpha} = 0$$

$$\tan\alpha - 2\tan^2\alpha = 0$$

$$\Leftrightarrow \tan\alpha(1-2\tan\alpha) = 0$$

$$\Leftrightarrow \tan\alpha = 0 \quad \text{ou} \quad \tan\alpha = \frac{1}{2}$$

$$\alpha = k\cdot\pi \quad \text{ou} \quad \alpha = \arctan\left(\frac{1}{2}\right) + k\cdot\pi$$

c) $t = \pm\frac{\pi}{2} + k2\pi$ donne $1 = 0$

On divise l'équation par $\cos^2 t$:

$$\tan^2 t - 4\tan t + 3 = 0$$

$$x^2 - 4x + 3 = 0 \Leftrightarrow (x-1)(x-3) = 0$$

$$\tan t = 1 \quad \text{ou} \quad \tan t = 3$$

$$t = \arctan(1) + k \cdot \pi = \frac{\pi}{4} + k \cdot \pi$$

$$t = \arctan(3) + k \cdot \pi$$

d) $1 - 2\sin x \cos x - 2\cos^2 x = 0 \quad \left(x \neq \pm\frac{\pi}{2} + k \cdot 2\pi\right)$

$$\underbrace{\cos^2 x + \sin^2 x}_{=1} - 2\sin x \cos x - 2\cos^2 x = 0$$

$$1 + \tan^2 x - 2\tan x - 2 = 0$$

$$\tan^2 x - 2\tan x - 1 = 0$$

$$t^2 - 2t - 1 = 0$$

$$t = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$$x = \arctan(1 \pm \sqrt{2}) + k \cdot \pi$$

e) $1 + 4\tan\varphi - 5\tan^2\varphi = 0$

$5\tan^2\varphi - 4\tan\varphi - 1 = 0$

$$5x^2 - 4x - 1 = 0 \iff x = \frac{4 \pm \sqrt{16+20}}{10} \begin{cases} 1 \\ -\frac{1}{5} \end{cases}$$

$$\varphi = \frac{\pi}{4} + k\cdot\pi \;\Big/\; \varphi = \arctan\left(-\frac{1}{5}\right) + k\cdot\pi$$

f) $\sin 2x = 2\sin x\cos x \Rightarrow \sin x\cos x = \frac{1}{2}\sin 2x$

$$5\cdot\sin^2(2t) + \frac{3}{2}\sin(2t) - 4 = 0$$

$$5x^2 + \frac{3}{2}x - 4 = 0$$

$$10x^2 + 3x - 8 = 0$$

$$x = \frac{-3 \pm \sqrt{9+320}}{20}$$

$$x \simeq 0{,}7569$$

$$2t = \arcsin(0{,}7569) + k \cdot 2\pi$$

$$t = \frac{1}{2}\arcsin(0{,}7569) + k \cdot \pi$$

$$2t = \pi - \arcsin(0{,}7569) + k \cdot 2\pi$$

$$t = \frac{\pi}{2} - \frac{1}{2}\arcsin(0{,}7569) + k \cdot \pi$$