a) $\vec{AB} \perp \vec{AC} \Leftrightarrow \vec{AB} \cdot \vec{AC} = 0$

$$\vec{AB} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} \lambda + 2 \\ \lambda - 4 \end{pmatrix}$$

$$\vec{AB} \cdot \vec{AC} = 3(\lambda + 2) - 6(\lambda - 4)$$

$$= 3\lambda + 6 - 6\lambda + 24$$

$$= -3\lambda + 30 = 0 \Leftrightarrow \boxed{\lambda = 10}$$

b) $\vec{AB} \perp \vec{BC} \Leftrightarrow \vec{AB} \cdot \vec{BC} = 0$

$$\vec{BC} = \begin{pmatrix} \lambda - 1 \\ \lambda + 2 \end{pmatrix}$$

$$\vec{AB} \cdot \vec{BC} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} \lambda - 1 \\ \lambda + 2 \end{pmatrix}$$

$$= 3(\lambda - 1) - 6(\lambda + 2)$$

$$= 3\lambda - 3 - 6\lambda - 12$$

$$= -3\lambda - 15 = 0 \Leftrightarrow \boxed{\lambda = -5}$$

c) $\vec{AC} \perp \vec{BC} \Leftrightarrow \vec{AC} \cdot \vec{BC} = 0$

$$\vec{AC} = \begin{pmatrix} \lambda + 2 \\ \lambda - 4 \end{pmatrix} \qquad \vec{BC} = \begin{pmatrix} \lambda - 1 \\ \lambda + 2 \end{pmatrix}$$

$$\begin{pmatrix} \lambda + 2 \\ \lambda - 4 \end{pmatrix} \cdot \begin{pmatrix} \lambda - 1 \\ \lambda + 2 \end{pmatrix} = (\lambda + 2)(\lambda - 1) + (\lambda - 4)(\lambda + 2)$$

$$= \lambda^2 + \lambda - 2 + \lambda^2 - 2\lambda - 8$$

$$= 2\lambda^2 - \lambda - 10 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1 + 80}}{4} = \begin{cases} \dfrac{1+9}{4} = \boxed{25} \\ \dfrac{1-9}{4} = \boxed{-2} \end{cases}$$