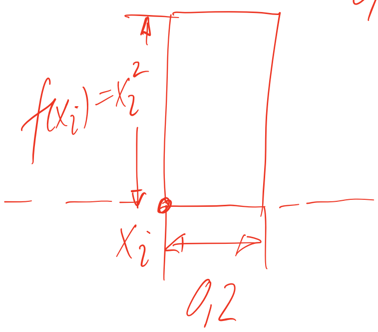
a) $u_5 = 0 + 0{,}2^2 \cdot 0{,}2 + 0{,}4^2 \cdot 0{,}2 + 0{,}6^2 \cdot 0{,}2 + 0{,}8^2 \cdot 0{,}2$

$= 0{,}2\left(0{,}2^2 + 0{,}4^2 + 0{,}6^2 + 0{,}8^2\right)$

base $= 0{,}4 - 0{,}2$
$= 0{,}2$

$= 0{,}24$



hauteur ($x_i = 0{,}2$ / $f(x_i) = x_i^2 = 0{,}2^2$)

$v_5 = 0{,}2^2 \cdot 0{,}2 + 0{,}4^2 \cdot 0{,}2 + 0{,}6^2 \cdot 0{,}2 + 0{,}8^2 \cdot 0{,}2 + 1^2 \cdot 0{,}2$

$= 0{,}2\left(0{,}2^2 + 0{,}4^2 + 0{,}6^2 + 0{,}8^2 + 1\right) = 0{,}44$

b) $u_{10} = 0 + 0{,}1^2 \cdot 0{,}1 + 0{,}2^2 \cdot 0{,}1 + 0{,}3^2 \cdot 0{,}1 + \dots + 0{,}9^2 \cdot 0{,}1$

$= 0{,}29 = 0{,}1\left(0{,}1^2 + 0{,}2^2 + \dots + 0{,}9^2\right)$

$v_{10} = 0{,}1^2 \cdot 0{,}1 + 0{,}2^2 \cdot 0{,}1 + 0{,}3^2 \cdot 0{,}1 + \dots + 1^2 \cdot 0{,}1$

$= 0{,}39 = 0{,}1\left(0{,}1^2 + 0{,}2^2 + \dots + 1^2\right)$

c) $u_n = 0 + \left(\frac{1}{n}\right)^2 \cdot \frac{1}{n} + \left(\frac{2}{n}\right)^2 \cdot \frac{1}{n} + \dots + \left(\frac{n-1}{n}\right)^2 \cdot \frac{1}{n}$

$= \frac{1}{n} \cdot \left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n-1}{n}\right)^2\right]$

$$v_n = \left(\frac{1}{n}\right)^2 \cdot \frac{1}{n} + \left(\frac{2}{n}\right)^2 \cdot \frac{1}{n} + \dots + \left(\frac{n}{n}\right)^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n-1}{n}\right)^2 + \left(\frac{n}{n}\right)^2\right]$$

Tables numériques p. 15, 2ème formule

On peut écrire:

$$u_n = \frac{1}{n} \cdot \frac{1^2 + 2^2 + \dots + (n-1)^2}{n^2} = \frac{1}{\cancel{n}} \cdot \frac{\frac{1}{6}(n-1)\cancel{n}(2n-1)}{n^2}$$

$$= \frac{1}{6} \cdot \frac{(n-1)(2n-1)}{n^2} = \frac{2n^2 - 3n + 1}{6n^2}$$

idem

$$v_n = \frac{1}{n} \cdot \frac{1^2 + 2^2 + \dots + n^2}{n^2} = \frac{1}{\cancel{n}} \cdot \frac{\frac{1}{6}\cancel{n}(n+1)(2n+1)}{n^2}$$

$$= \frac{1}{6} \frac{(n+1)(2n+1)}{n^2} = \frac{2n^2 + 3n + 1}{6n^2}$$

On peut donc écrire, pour tout $n \geq 1$:

$$\frac{2n^2-3n+1}{6n^2} < \int_0^1 x^2\,dx < \frac{2n^2+3n+1}{6n^2}$$

$$\begin{array}{ccc} \Big\downarrow {\scriptstyle n\to\infty} & \Big\downarrow & \Big\downarrow {\scriptstyle n\to\infty} \\ 1/3 & 1/3 & 1/3 \end{array}$$

Pas de choix

On peut vérifier le résultat :

$$\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$