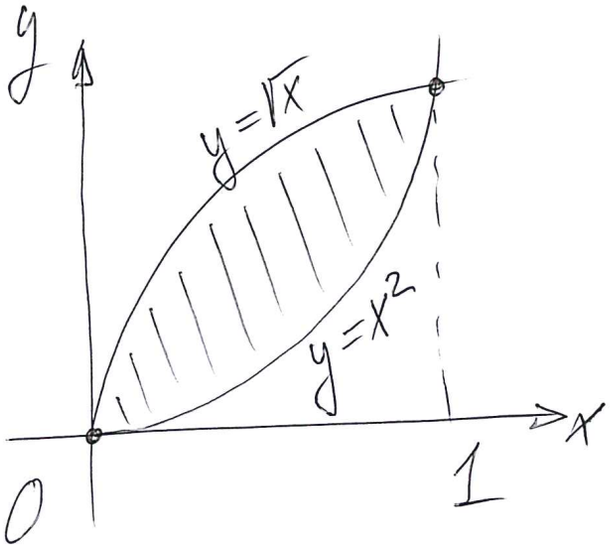
a) $\sqrt{x} = x^2 \iff x = x^4 \iff x(x^3 - 1) = 0$

$\iff x = 0 \,/\, x = 1$



$$V = \pi \int_0^1 (\sqrt{x})^2 dx - \pi \int_0^1 (x^2)^2 dx$$

$$\int_0^1 x\,dx = \frac{1}{2}x^2 \Big|_0^1 = \frac{1}{2}$$

$$\int_0^1 x^4 dx = \frac{1}{5}x^5 \Big|_0^1 = \frac{1}{5}$$

$$\Rightarrow V = \pi \cdot \left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$$

b) $f(x) = g(x) \Leftrightarrow x^2 - 2x + 6 = -x^2 + 10$

$\Leftrightarrow 2x^2 - 2x - 4 = 0 \Leftrightarrow x^2 - x - 2 = 0$

$\Leftrightarrow x = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

$$V = \pi \left| \left(\int_{-1}^{2} f^2(x)\,dx - \int_{-1}^{2} g^2(x)\,dx \right) \right|$$

$$f^2(x) = x^4 + 4x^2 + 36 - 4x^3 - 24x + 12x^2$$

$$= x^4 - 4x^3 + 16x^2 - 24x + 36$$

$$g^2(x) = x^4 - 20x^2 + 100$$

$$\int f^2(x)\,dx = \frac{x^5}{5} - x^4 + \frac{16x^3}{3} - 12x^2 + 36x + c$$

$$\int g^2(x)\,dx = \frac{x^5}{5} - \frac{20x^3}{3} + 100x + c$$

b) $\int_{-1}^{2} f^2(x)\,dx - \int_{-1}^{2} g^2(x)\,dx$

$= \int_{-1}^{2} \left(f^2(x) - g^2(x)\right) dx$

$= -x^4 + \frac{36}{3}x^3 - 12x^2 - 64x \Big|_{-1}^{2}$

$= \underbrace{-16 + 96 - 48 - 128}_{-96} - \underbrace{(-1 - 12 - 12 + 64)}_{39}$

$= -135$

$\Rightarrow V = 135\pi$