a) $$\int_1^2 e^x\,dx = e^x \Big|_1^2 = e^2 - e^1 = e^2 - e$$

b) $$\int_1^2 e^{3x-7}\,dx = \frac{1}{3}\int_1^2 e^{3x-7}\cdot 3\cdot dx$$

$$= \frac{1}{3} e^{3x-7}\Big|_1^2 = \frac{1}{3}\left(e^{3\cdot 2-7} - e^{3\cdot 1-7}\right)$$

$$= \frac{1}{3}\left(e^{-1} - e^{-4}\right) = \frac{1}{3}\left(\frac{1}{e} - \frac{1}{e^4}\right)$$

c) $$\int_0^2 e^{x^2}\cdot x\cdot dx = \frac{1}{2}\int_0^2 e^{x^2}\cdot 2x\cdot dx$$

$$= \frac{1}{2} e^{x^2}\Big|_0^2 = \frac{1}{2}\left(e^4 - e^0\right) = \frac{1}{2}\left(e^4 - 1\right)$$

d) $2 \cdot \int_1^2 \frac{1}{e^{\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} \cdot dx =$

$$-2 \cdot \int_1^2 e^{-\sqrt{x}} \cdot \frac{-1}{2\sqrt{x}}\, dx =$$

$$(-\sqrt{x})' = -\left(x^{\frac{1}{2}}\right)' = -\frac{1}{2} x^{\frac{1}{2}-1} = -\frac{1}{2} x^{-\frac{1}{2}} = \frac{-1}{2\sqrt{x}}$$

$$-2 \cdot e^{-\sqrt{x}} \Big|_1^2 = -2\left(e^{-\sqrt{2}} - e^{-\sqrt{1}}\right) =$$

$$-2\left(\frac{1}{e^{\sqrt{2}}} - \frac{1}{e}\right)$$

e) $(f \cdot g)' = f' \cdot g + f \cdot g'$

$(x \cdot e^x)' = 1 \cdot e^x + x \cdot e^x$

$$\Rightarrow \quad x \cdot e^x = \int 1 \cdot e^x dx + \int x \cdot e^x dx$$

$$\Leftrightarrow \int x e^x \cdot dx = x e^x - \int e^x dx$$

$$= x e^x - e^x$$

On a donc

$$\int_0^1 x e^x dx = (x e^x - e^x)\Big|_0^1$$

$$= (1 e^1 - e^1) - (0 \cdot e^0 - e^0) = 1$$

f) $(f \cdot g)' = f' \cdot g + f \cdot g'$

$$(x^2 \cdot e^x)' = 2x \cdot e^x + x^2 \cdot e^x$$

$$\Rightarrow \quad x^2 \cdot e^x = \int 2x e^x \, dx + \int x^2 \cdot e^x \, dx$$

$$\Leftrightarrow \quad x^2 \cdot e^x = 2 \int x e^x \, dx + \int x^2 e^x \, dx$$

On a vu à la question précédente que

$$\int x e^x \, dx = x e^x - e^x$$

On a donc

$$x^2 \cdot e^x = 2x e^x - 2e^x + \int x^2 e^x \, dx$$

Ainsi, $\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x$

Ce qui fait que

$$\int_1^{\ln(2)} x^2 e^x dx = \left(x^2 e^x - 2xe^x + 2e^x\right)\Big|_1^{\ln(2)}$$

$$= \ln(2)^2 \cdot 2 - 2\ln(2) \cdot 2 + 2 \cdot 2 - (e - 2e + 2e)$$

$$= 2\ln(2)^2 - 4\ln(2) + 4 - e$$