$h : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ $\quad h$ linéaire

$H = \begin{pmatrix} 0{,}6 & 0{,}8 \\ 0{,}8 & -0{,}6 \end{pmatrix}$ relativement à $\mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$

$$\boxed{\mathcal{B}' = \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right)}$$

$$\mathbb{R}^2_{\mathcal{B}} \xrightarrow{H} \mathbb{R}^2_{\mathcal{B}}$$

$$\begin{tikzcd}
\mathbb{R}^2_{\mathcal{B}} \arrow[r, "H"] \arrow[d, "P^{-1}"] & \mathbb{R}^2_{\mathcal{B}} \\
\mathbb{R}^2_{\mathcal{B}'} \arrow[u, "P"] \arrow[r, "H'"'] & \mathbb{R}^2_{\mathcal{B}'}
\end{tikzcd}$$

Inverse de la matriz P

$$P = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\boxed{H' = P^{-1} \cdot H \cdot P}$$

$$\left(\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 2 & -1 & 1 & 0 \end{array}\right)$$

$L_2 \leftarrow L_2 - 2 \cdot L_1$ $\qquad$ $L_2 \leftarrow -\frac{1}{5} L_2$

$$\sim \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -5 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & 1 & -0{,}2 & 0{,}4 \end{array}\right)$$

$L_1 \leftarrow L_1 - 2 L_2$

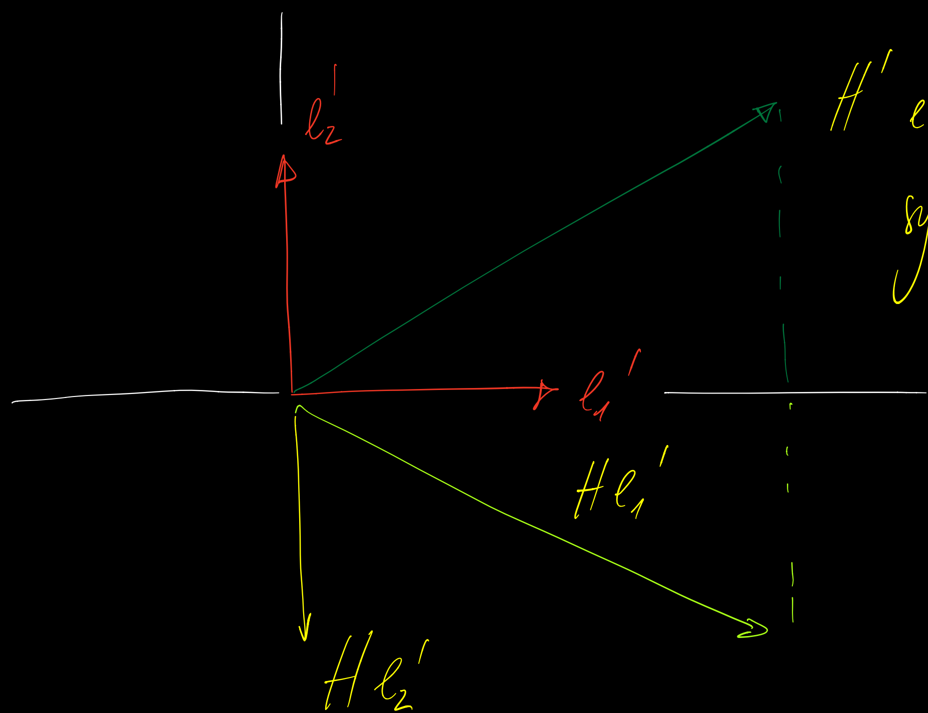
$$\sim \left(\begin{array}{cc|cc} 1 & 0 & 0{,}4 & 0{,}2 \\ 0 & 1 & -0{,}2 & 0{,}4 \end{array}\right) \Rightarrow \underline{P}^{-1} = \begin{pmatrix} 0{,}4 & 0{,}2 \\ -0{,}2 & 0{,}4 \end{pmatrix}$$

$$H' = P^{-1} H P$$

$$= \begin{pmatrix} 0{,}4 & 0{,}2 \\ -0{,}2 & 0{,}4 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} \cancel{0{,}6} & \cancel{0{,}8} \\ 0{,}8 & -0{,}6 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}}$$

$$= \begin{pmatrix} 0{,}4 & 0{,}2 \\ -0{,}2 & 0{,}4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \quad H' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



H' est une symétrie axiale d'axe « $k \cdot e_1'$ »