c) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

$n=1$ $\quad \sum_{k=1}^{1} 1^3 = 1 = \frac{1^2(1+1)^2}{4} = \frac{4}{4}$ ✓

n ✓ $\Rightarrow$ $n+1$ ✓

$$\sum_{k=1}^{n+1} k^3 = \sum_{k=1}^{n} k^3 + (n+1)^3$$

hyp. de rec. ↓

$$= \frac{n^2(n+1)^2}{4} + \frac{4(n+1)^3}{4}$$

$$= \frac{(n+1)^2\left(n^2 + 4(n+1)\right)}{4}$$

$$= \frac{(n+1)^2(n^2+4n+4)}{4} = \frac{(n+1)^2(n+2)^2}{4}$$

$$= \frac{(n+1)^2 \cdot (n+1+1)^2}{4}$$

CQFD