f) $n=1$

$$\sum_{k=1}^{1} \frac{k^2}{(2k-1)(2k+1)} = \frac{1}{1\cdot 3}$$

$$= \frac{1(1+1)}{2(2\cdot 1+1)} = \frac{1\cdot 2}{2\cdot 3}$$

n ✓ $\Rightarrow$ $n+1$ ✓

$$\sum_{k=1}^{n+1} \frac{k^2}{(2k-1)(2k+1)}$$

hyp. de rec

$$= \frac{n(n+1)}{2(2n+1)} + \frac{(n+1)^2}{(2n+1)(2n+3)}$$

$$= \frac{n\cdot(n+1)\cdot(2n+3) + 2(n+1)^2}{2(2n+1)(2n+3)}$$

$$= \frac{(n+1)(2n^2+3n+2n+2)}{2(2n+1)(2n+3)}$$

$$= \frac{(n+1)(2n+1)(n+2)}{2(2n+1)(2n+3)}$$

$$= \frac{(n+1)(n+1+1)}{2(2(n+1)+1)}$$

CQFD