

1.1.9

3MS

$$2) \lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = \ll \frac{\ln(0+1)}{0} \gg = \ll \frac{0}{0} \gg = \text{IND.}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(\ln(x+1))'}{x'} &= \lim_{x \rightarrow 0} \frac{\frac{1}{x+1}}{1} \\ &= \lim_{x \rightarrow 0} \frac{1}{x+1} = \ll \frac{1}{0+1} \gg \\ &= 1 \end{aligned}$$

B.-H.

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\ln(x+1)}{x} = 1$$

$$6) \lim_{x \rightarrow 0} \frac{\ln(\cos(x))}{x^2} = \ll \frac{\ln(\cos(0))}{0^2} \gg = \ll \frac{0}{0} \gg = \text{IND.}$$

$$\lim_{x \rightarrow 0} \frac{(\ln(\cos(x)))'}{(x^2)'} = \lim_{x \rightarrow 0} \frac{\frac{u'}{u}}{2x} = \lim_{x \rightarrow 0} \frac{\frac{-\sin(x)}{\cos(x)}}{2x}$$

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$$= \lim_{x \rightarrow 0} \frac{-\tan(x)}{2x} = \ll \frac{-\tan(0)}{2 \cdot 0} \gg = \ll \frac{0}{0} \gg$$

IND.

$$\lim_{x \rightarrow 0} \frac{(-\tan(x))'}{(2x)'} = \lim_{x \rightarrow 0} \frac{-\frac{1}{\cos^2(x)}}{2}$$

$$= \lim_{x \rightarrow 0} -\frac{1}{2\cos^2(x)} = \ll -\frac{1}{2\cos^2(0)} \gg$$

$$= -\frac{1}{2} \quad \text{car } (\underbrace{\cos(0)}_1)^2 = 1$$

B.-H.

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\ln(\cos(x))}{x^2} = -\frac{1}{2}$$

$$c) \lim_{x \rightarrow e} \frac{\ln(x) - 1}{x - e} = \ll \frac{\overbrace{\ln(e)}^1 - 1}{e - e} \gg = \ll \frac{0}{0} \gg$$

IND.

$$\lim_{x \rightarrow e} \frac{(\ln(x) - 1)'}{(x - e)'} = \lim_{x \rightarrow e} \frac{\frac{1}{x} - 0}{1 - 0} = \lim_{x \rightarrow e} \frac{1}{x}$$

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$$= \ll \frac{1}{e} \gg = \frac{1}{e}$$

B.-H.

$$\Rightarrow \lim_{x \rightarrow e} \frac{\ln(x) - 1}{x - e} = \frac{1}{e}$$

$$d) \lim_{x \rightarrow 2} \frac{\ln(x^2 - 3)}{2 - x} = \ll \frac{\ln(4 - 3)}{2 - 2} \gg = \ll \frac{0}{0} \gg$$

ind.

$$\lim_{x \rightarrow 2} \frac{(\ln(x^2 - 3))'}{(2 - x)'} = \lim_{x \rightarrow 2} \frac{\frac{(x^2 - 3)'}{x^2 - 3}}{(2 - x)'}$$

$$= \lim_{x \rightarrow 2} \frac{\frac{2x}{x^2 - 3}}{0 - 1} = \lim_{x \rightarrow 2} - \frac{2x}{x^2 - 3}$$

$$= \ll - \frac{2 \cdot 2}{4 - 3} \gg = -4$$

B.-H.

$$\Rightarrow \lim_{x \rightarrow 2} \frac{\ln(x^2 - 3)}{2 - x} = -4$$

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$$e) \lim_{x \rightarrow -\infty} \frac{\ln(x^2+1)}{x} = \ll \frac{\overbrace{\ln((- \infty)^2+1)}^{+\infty}}{-\infty} \gg = \ll \frac{+\infty}{-\infty} \gg = \text{IND.}$$

$$\lim_{x \rightarrow -\infty} \frac{(\ln(x^2+1))'}{(x)'} = \lim_{x \rightarrow -\infty} \frac{\frac{(x^2+1)'}{x^2+1}}{1}$$

$$= \lim_{x \rightarrow -\infty} \frac{2x}{x^2+1} = \lim_{x \rightarrow -\infty} \frac{2x}{x^2} = \lim_{x \rightarrow -\infty} \frac{2}{x}$$

On peut négliger cela $= \ll \frac{2}{-\infty} \gg = 0$

B.-H.

$$\Rightarrow \lim_{x \rightarrow -\infty} \frac{\ln(x^2+1)}{x} = 0$$

$$f) \lim_{x \rightarrow +\infty} \frac{\ln(x)+1}{1-\ln(x)} = \ll \frac{+\infty+1}{1-\infty} \gg = \ll \frac{+\infty}{-\infty} \gg = \text{IND.}$$

$$\lim_{x \rightarrow +\infty} \frac{(\ln(x)+1)'}{(1-\ln(x))'} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x}+0}{0-\frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{1/x}{-1/x}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{x} \cdot \frac{-x}{1} = \lim_{x \rightarrow +\infty} -1 = -1$$

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Notation: $\ln^2(x) = (\ln(x))^2$

B.-H.

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{\ln(x)+1}{1-\ln(x)} = -1$$

$$g) \lim_{x \rightarrow +\infty} \frac{\ln(x^2)}{\ln^2(x)} = \ll \frac{\ln((+\infty)^2)}{\ln^2(+\infty)} \gg = \ll \frac{+\infty}{+\infty} \gg$$

$$\lim_{x \rightarrow +\infty} \frac{(\ln(x^2))'}{[(\ln(x))^2]'} = \lim_{x \rightarrow +\infty} \frac{\frac{(x^2)'}{x^2}}{2 \ln(x) \cdot \frac{1}{x}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\frac{2x}{x^2}}{2 \ln(x) \cdot \frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{2x}{x^2 \cdot 2 \cdot \ln(x)} \cdot \frac{x}{1}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{\ln(x)} = \ll \frac{1}{\ln(+\infty)} \gg = \ll \frac{1}{+\infty} \gg = 0$$

B.-H.

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{\ln(x^2)}{\ln^2(x)} = 0$$

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$$h) \lim_{x \rightarrow +\infty} \frac{\ln(x)}{e^x} = \ll \frac{+\infty}{+\infty} \gg$$

INDÉTERMINÉ

$$\lim_{x \rightarrow +\infty} \frac{(\ln(x))'}{(e^x)'} = \lim_{x \rightarrow +\infty} \frac{1/x}{e^x}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{x} \cdot \frac{1}{e^x} = \lim_{x \rightarrow +\infty} \frac{1}{x \cdot e^x}$$

$$= \ll \frac{1}{+\infty \cdot +\infty} \gg = \ll \frac{1}{+\infty} \gg = 0$$

B.-H.

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{\ln(x)}{e^x} = 0$$