$\boxed{1.3.5}$

3MS

a) $\int \cos(3x)\,dx = \int \cos(3x)\cdot \overbrace{3\cdot\frac{1}{3}}^{1}\,dx$

$= \int \frac{1}{3}\cos(3x)\cdot 3\,dx = \frac{1}{3}\int \cos(3x)\cdot 3\,dx$

$= \frac{1}{3}\sin(3x) + c$

b) $\int \sin\left(2x-\frac{\pi}{3}\right)\cdot \overbrace{2\cdot\frac{1}{2}}^{1}\,dx =$

$\frac{1}{2}\int \sin\left(2x-\frac{\pi}{3}\right)\cdot 2\,dx = \frac{1}{2}\left(-\cos\left(2x-\frac{\pi}{3}\right)\right)+c$

$= -\frac{1}{2}\cos\left(2x-\frac{\pi}{3}\right)+c$

c) $\int (x+3)^3\,dx = \frac{1}{4}(x+3)^4 + C$

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d) $\displaystyle\int (2x-1)^2 dx = \int (2x-1)^2 \cdot 2 \cdot \frac{1}{2} dx$

$\displaystyle = \int \frac{1}{2} (2x-1)^2 \cdot 2 dx = \frac{1}{2} \int (2x-1)^2 \cdot 2 dx$

$\displaystyle = \frac{1}{2} \cdot \frac{1}{3} (2x-1)^3 + C = \frac{1}{6} (2x-1)^3 + C$

e) $\displaystyle\int (7x-2)^5 dx = \frac{1}{7} \int (7x-2)^5 \cdot 7 dx$

$\displaystyle = \frac{1}{7} \cdot \frac{1}{6} (7x-2)^6 + C = \frac{1}{42} \cdot (7x-2)^6$

f) $\displaystyle\int (3x^2+x)^3 (6x+1) dx$

On note que $(3x^2+x)' = 3 \cdot 2x + 1 = 6x+1$.

On a donc $\left[(3x^2+4)^4\right]' = 4(3x^2+4)^3 \cdot (6x+1)$

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On en déduit: $\left[\frac{1}{4}(3x^2+4)^4\right]' = (3x^2+4)^3\cdot(6x+1)$

Ainsi, $\displaystyle\int (3x^2+x)^3\,(6x+1)\,dx = \int \left[\frac{1}{4}(3x^2+4)^4\right]' dx$

$= \frac{1}{4}(3x^2+4)^4 + C$

g) $\displaystyle\int (4x^2+3)^4\cdot x\cdot dx = \int (4x^2+3)^4\cdot 8\cdot x\cdot\frac{1}{8}\cdot dx$

$(4x^2+3)' = 4\cdot 2x + 0 = 8x$

$= \displaystyle\frac{1}{8}\int (4x^2+3)^4\,(4x^2+3)'\,dx = \frac{1}{8}\cdot\frac{1}{5}\cdot(4x^2+3)^5 + C$

$= \displaystyle\frac{1}{40}\cdot(4x^2+3)^5 + C$

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h) $\int \sin^2(x)\cos(x)\,dx = \int [\sin(x)]^2 \cdot (\sin(x))'\,dx$

$= \frac{1}{3}(\sin(x))^3 + C$

i) $\int (\tan(x))^2 \cdot \frac{1}{\cos^2(x)}\,dx = \int (\tan(x))^2 \cdot (\tan(x))'\,dx$

$= \frac{1}{3}(\tan(x))^3 + C$

j) $\int \sqrt{x+3}\,dx = \int (x+3)^{1/2} \cdot 1 \cdot dx$

$= \frac{1}{1/2+1}(x+3)^{1/2+1} + C = \frac{2}{3}(x+3)^{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{(x+3)^3} + C$

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k) $\displaystyle\int \frac{dx}{\sqrt{3x+1}} = \int \frac{1}{\sqrt{3x+1}}\,dx$

$\displaystyle = \int \frac{1}{(3x+1)^{1/2}}\,dx = \int (3x+1)^{-1/2}\,dx$

$\displaystyle = \frac{1}{3}\int (3x+1)^{-1/2}\cdot 3\cdot dx = \frac{1}{3}\int (3x+1)^{-1/2}\cdot(3x+1)'\,dx$

$\displaystyle = \frac{1}{3}\cdot\frac{1}{-1/2+1}(3x+1)^{-1/2+1} + C$

$\displaystyle = \frac{1}{3}\cdot 2\cdot(3x+1)^{1/2} + C = \frac{2}{3}\sqrt{3x+1} + C$

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e) $\int \frac{1}{\sqrt{x^2+2x}} \cdot (x+1) \cdot dx =$

$\int (x^2+2x)^{-1/2} \cdot (x+1) \cdot dx = \frac{1}{2} \int (x^2+2x)^{-1/2} \cdot (2x+2) \cdot dx$

$= \frac{1}{2} \int (x^2+2x)^{-1/2} \cdot \underbrace{(x^2+2x)'}_{} \cdot dx$

$= \frac{1}{2} \frac{1}{1+(-1/2)} (x^2+2x)^{1+(-1/2)} + C$

$= (x^2+2x)^{1/2} + C = \sqrt{x^2+2x} + C$