$\boxed{91 = 7 \cdot 13}$ $\boxed{17 = 17 \cdot 1}$

$\boxed{4 = 2 \cdot 2}$

$21 = 7 \cdot 3$

$100 = 25 \cdot 4 = 5^2 \cdot$

$1283751362187621671335217171717 \stackrel{?}{=} a \cdot b$

$a, b \in \mathbb{N}$

$\boxed{\text{Nombres entiers}}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

est un diviseur de b

$\boxed{a, b \in \mathbb{Z} \qquad a \mid b \quad \text{si} \quad \exists\, z \in \mathbb{Z} \quad \text{tq.} \quad a \cdot z = b}$

appartient à — divise — il existe — tel que

$-4, -3 \qquad -4 \nmid -3$

Exemple : $3 \mid 9$ $\quad 9 = 3 \cdot \underset{z}{3}$ $\qquad -8 \mid 48 \qquad -1 \mid -2$

$25 \mid 100$ $\quad 100 = 25 \cdot \underset{z}{4}$ $\qquad 3 \mid -9 \qquad -2 \mid -8$

$5 \nmid 17 \qquad 8 \nmid 1$

$\boxed{4.2.1}$ <u>Déf.</u> $a, b \in \mathbb{Z}$. $\boxed{a \mid b \Leftrightarrow \exists z \in \mathbb{Z} \text{ tq. } az = b}$

a) $a \mid a$ $\boxed{\text{car } a \cdot 1 = a. \quad (z = 1)}$ $\square$

$1 \mid a$ $\boxed{\text{car } 1 \cdot a = a. \quad (z = a)}$ $\square$ $\quad \forall a \in \mathbb{Z}$

$a \mid 0$ $\boxed{\text{car } a \cdot 0 = 0 \quad (z = 0)}$ $\square$

b) $0 \mid a \Leftrightarrow a = 0$

$\Rightarrow$

$\Leftarrow$

$\boxed{\Rightarrow}$ $0 \mid a \Rightarrow \exists z \in \mathbb{Z}$ tq. $\overbrace{0 \cdot z}^{0} = a \Rightarrow 0 = a \Rightarrow a = 0$ $\square$

$\boxed{\Leftarrow}$ $a = 0 \Rightarrow 0 \cdot z = a \quad \forall z \in \mathbb{Z} \Rightarrow 0 \mid a$ $\square$

pour tout

c) $\boxed{a \mid b} \Rightarrow \boxed{-a \mid b}$

$a \mid b \Rightarrow \exists z \in \mathbb{Z}$ tq $az = b \Rightarrow \overbrace{(-1) \cdot (-1)}^{1} \cdot a \cdot z = b$

$\Rightarrow (-a) \cdot (-z) = b$ Or $\boxed{-z \in \mathbb{Z}}$. Soit $w = -z$.

On peut écrire $(-a)\cdot w = b$ avec $w \in \mathbb{Z} \Rightarrow -a \mid b$ $\square$

$$\boxed{\begin{matrix} P \Rightarrow Q & & Q \Rightarrow R \\ & \text{et} & \\ Q \Rightarrow P & & R \Rightarrow Q \end{matrix}} \Leftrightarrow \boxed{P \Leftrightarrow Q \Leftrightarrow R}$$

$$\Leftrightarrow \boxed{\begin{tikzcd} P \arrow[r, Rightarrow] & Q \arrow[d, Rightarrow] \\ & R \arrow[ul, Rightarrow] \end{tikzcd}}$$

$$\begin{tikzcd} \boxed{a \mid b} \arrow[r, Rightarrow] & \boxed{-a \mid b} \arrow[dl, Rightarrow] \\ \boxed{a \mid -b} \arrow[u, Rightarrow] & \end{tikzcd}$$

c) $a \mid b$ $\quad \boxed{\exists z \in \mathbb{Z} \text{ tq. } a \cdot z = b}$ $\Rightarrow |a \cdot z| = |b|$

$\Rightarrow |a| \cdot |z| = |b|$

$\Rightarrow |a| \mid |b|$

$-a \mid b$ $\quad \exists w \in \mathbb{Z}$ tq. $(-a)\cdot w = b$

Posons $w = -z \in \mathbb{Z}$ $\Rightarrow (-a)(-z) = a \cdot z = b$

$\Rightarrow (-a) \cdot w = b$

$\Rightarrow -a \mid b$ $\square$

$\boxed{-a \mid b}$ $\quad \exists z \in \mathbb{Z}$ tq. $(-a)\cdot z = b \Rightarrow (-1)(-a)\cdot z = (-1)b$

$\Rightarrow a \cdot z = -b$

$\Rightarrow a \mid -b$