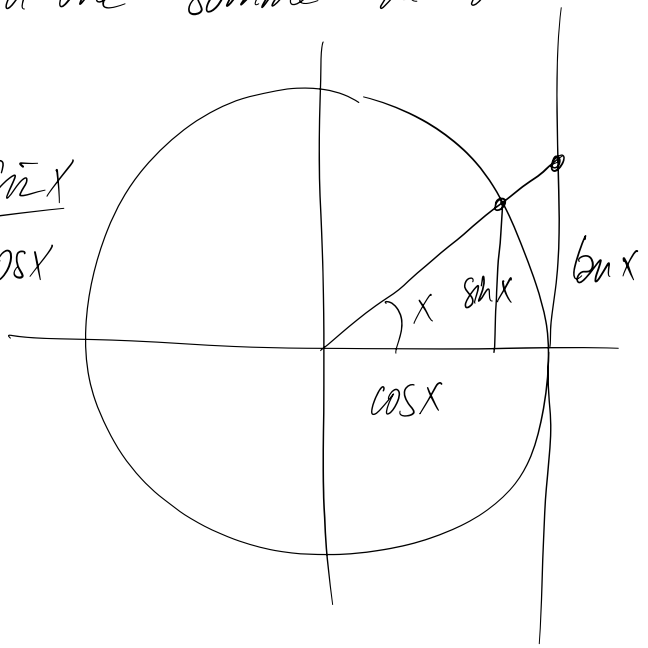
TRIGO

Tangente d'une somme de deux arcs.

$$\frac{\tan x}{1} = \frac{\sin x}{\cos x} \iff \tan x = \frac{\sin x}{\cos x}$$



$$\cos^2 x + \sin^2 x = 1$$

On a vu que :

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

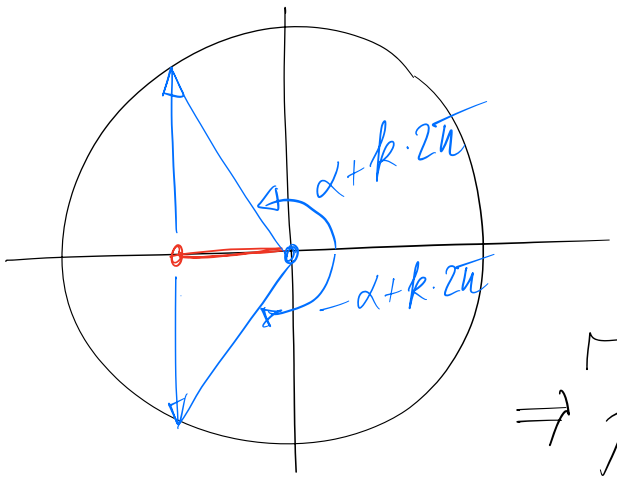
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\Rightarrow \tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}$$

$$\iff \tan(x+y) = \frac{(\sin x \cos y + \sin y \cos x) \cdot \frac{1}{\cos x \cos y}}{(\cos x \cos y - \sin x \sin y) \cdot \frac{1}{\cos x \cos y}}$$

$$\Leftrightarrow \quad \tan(x+y) = \frac{\tan x \left[\frac{\sin x}{\cos x} \cdot \frac{\cos y}{\cos y}\right]_{1} + \left[\frac{\sin y}{\cos y} \cdot \frac{\cos x}{\cos x}\right]_{1} \tan y}{\left[\frac{\cos x}{\cos x} \cdot \frac{\cos y}{\cos y}\right]_{1} - \left[\frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}\right]_{\tan x} \tan y}$$

$$\Leftrightarrow \quad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$



$$\cos(f(t)) = 2$$

$$\alpha = \arccos(2)$$

$$\Rightarrow \boxed{\begin{aligned} f(t) &= \alpha + k 2\pi \\ f(t) &= -\alpha + k 2\pi \end{aligned}}$$

donne t

4.3.4 g) $\tan(3t) = \cot(t)$

Vu que $\cot(t) = \dfrac{1}{\tan(t)}$ par définition, il faut résoudre

$$\tan(3t) = \frac{1}{\tan(t)}$$

Si $t \neq 0 + k \cdot \pi$, cela revient à résoudre

$$\tan(3t) \cdot \tan t = 1$$

$$\Leftrightarrow \tan(2t + t) \cdot \tan t = 1$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\Leftrightarrow \frac{\tan 2t + \tan t}{1 - \tan 2t \tan t} \cdot \tan t = 1$$

$$\Leftrightarrow (\tan 2t + \tan t) \cdot \tan t = 1 - \tan 2t \cdot \tan t$$

$$\Leftrightarrow \quad \tan 2t \cdot \tan t + \tan^2 t = 1 - \tan 2t \cdot \tan t$$

$$\Leftrightarrow \quad 2 \cdot \tan 2t \cdot \tan t + \tan^2 t = 1$$

$$\tan(t+t) = \frac{\tan t + \tan t}{1 - \tan t \cdot \tan t}$$

$$\Leftrightarrow \quad 2 \cdot \frac{2\tan t}{1-\tan^2 t} \cdot \tan t + \tan^2 t = 1$$

$$\Leftrightarrow \quad \frac{4\tan^2 t}{1-\tan^2 t} + \tan^2 t = 1$$

$$\Leftrightarrow \quad 4\tan^2 t + \tan^2 t - \tan^4 t = 1 - \tan^2 t$$

$$\Leftrightarrow \quad \tan^4 t - 6\tan^2 t + 1 = 0$$

Soit $x = \tan^2 t$

$$x^2 - 6x + 1 = 0 \quad \Leftrightarrow x = \frac{6 \pm \sqrt{36-4}}{2}$$

$$\Leftrightarrow \quad x = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$$\Leftrightarrow \quad \tan^2 t = 3 \pm 2\sqrt{2}$$

$$\Leftrightarrow \quad \tan t = \pm\sqrt{3 \pm 2\sqrt{2}}$$

$$\Leftrightarrow \quad t = \pm\frac{3\pi}{8} + k \cdot \pi \qquad \left(\pm\sqrt{3+2\sqrt{2}}\right)$$

$$t = \pm\frac{\pi}{8} + k \cdot \pi \qquad \left(\pm\sqrt{3-2\sqrt{2}}\right)$$