$$\frac{3\sin x}{2+\cos x} = f(x) = \frac{u}{v}$$

ÉTUDE SUR $[0; 2\pi]$

1.1 $$f'(x) = \frac{(3\sin x)' \cdot (2+\cos x) - 3\sin x \cdot (2+\cos x)'}{(2+\cos x)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$= \frac{3\cos x\,(2+\cos x) - 3\sin x \cdot (-\sin x)}{(2+\cos x)^2}$$

$$= \frac{6\cos x + 3\cos^2 x + 3\sin^2 x}{(2+\cos x)^2} = \frac{6\cos x + 3(\sin^2 x + \cos^2 x)}{(2+\cos x)^2}$$

$$= \frac{6\cos x + 3}{(2+\cos x)^2}$$

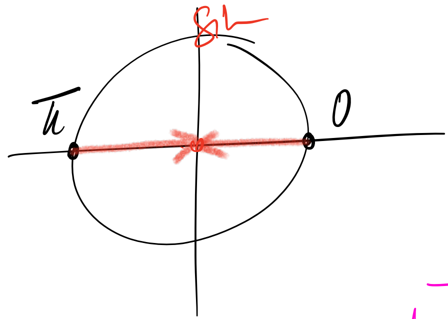
1.2 D_f $\quad f(x) = \frac{3\sin x}{2+\cos x}$

On a un problème si $\boxed{2+\cos x = 0}$ $\Leftrightarrow$ $\cos x = -2$
(division par 0)

$\alpha = \cos^{-1}(-2)$ impossible

$\Rightarrow D_f = \mathbb{R}$ (jamais de div. par 0)

Zeros: $\boxed{3\sin x = 0} \Leftrightarrow \sin x = 0$ sur $[0;2\pi]$



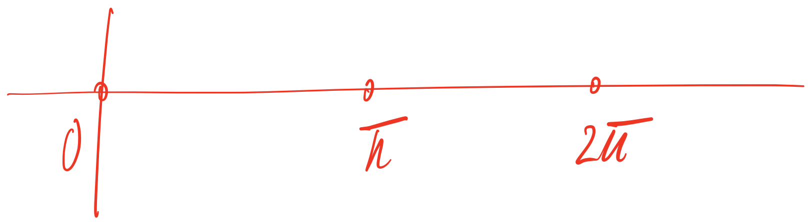
$x = 0 + k\cdot 2\pi$

$\pi - 0$

$x = \pi + k\cdot 2\pi$

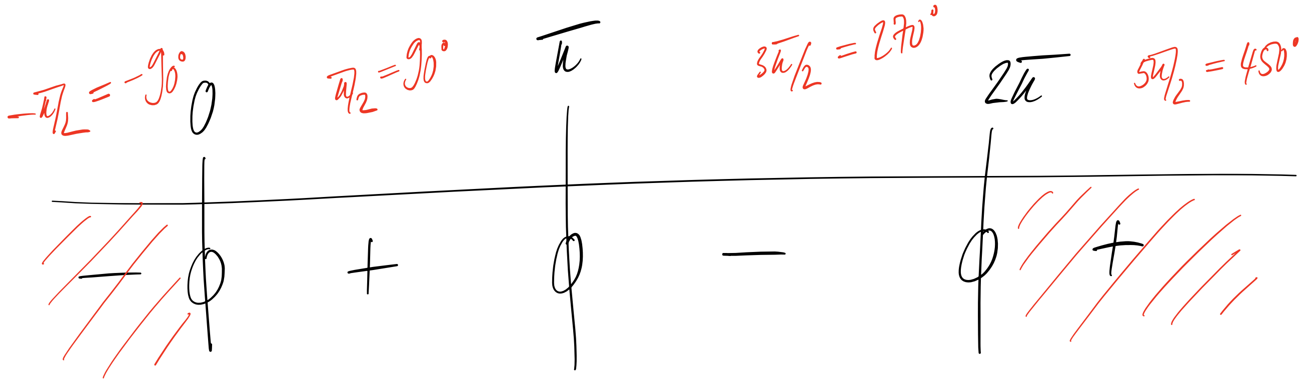
Finalement: $\boxed{x=0 \;/\; x=\pi \;/\; x=2\pi}$

$\alpha_{deg} = \frac{\alpha_{rad}}{\pi}\cdot 180°$



$f(x) = \frac{3\sin x}{2+\cos x}$

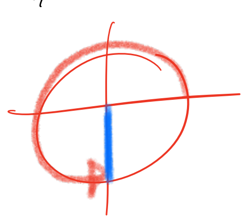
Signe :



$f(-\frac{\pi}{2}) = \frac{3\cdot(-1)}{2+0} = -1,5$

$f(\frac{3\pi}{2}) = \frac{3\cdot(-1)}{2} = -1,5$

$f(\frac{\pi}{2}) = \frac{3}{2} = 1,5$



$f(\frac{5\pi}{2}) = \frac{3\cdot 1}{2} = 1,5$

Croissance: $f'(x) = \dfrac{3+6\cos x}{(2+\cos x)^2}$

$D_{f'} = \mathbb{R}$ car $2+\cos x \neq 0$ si $x \in \mathbb{R}$

$f'(x) = 0 \iff \boxed{3+6\cos x = 0}$

$\iff 6\cos x = -3$

$\iff \cos x = -\dfrac{3}{6} = -\dfrac{1}{2}$ $\qquad \cos^{-1}\left(-\dfrac{1}{2}\right) = 120°$

$x = \pm \dfrac{2\pi}{3} + k\,2\pi$ $\qquad (\pm 120° + k \cdot 360°)$

$x = \underset{120°}{\dfrac{2\pi}{3}}$ / $x = -\dfrac{2\pi}{3} + \dfrac{6\pi}{3} = \underset{240°}{\dfrac{4\pi}{3}}$

0		$2\pi/3$	π	$4\pi/3$		2π
	+	0	−	0	+	

MAX $\left(\dfrac{2\pi}{3} ; ?\right)$ $\qquad$ MIN $\left(\dfrac{4\pi}{3} ; ?\right)$

$f'(x) = \dfrac{3+6\cos x}{(2+\cos x)^2}$ $\qquad f'(0) = \dfrac{9}{9} = 1 > 0$

$f'(2\pi) = f'(0)$

$f'(\pi) = \dfrac{-3}{1} < 0$

$f\left(\dfrac{2\pi}{3}\right) = \dfrac{3\sin\frac{2\pi}{3}}{2+\cos\frac{2\pi}{3}} \simeq 1{,}732$ $\qquad f\left(\dfrac{4\pi}{3}\right) = \dfrac{3\sin\frac{4\pi}{3}}{2+\cos\frac{2\pi}{3}} \simeq -1{,}732$

$MAX\left(\frac{2\pi}{3}; 1{,}732\right)$ $\qquad$ $MIN\left(\frac{4\pi}{3}; -1{,}732\right)$