

TRIGONOMETRIE 2MS

1) Résoudre les équations :

a) $1 - 2\sin x = 0$ ✓ a') $1 - 3\sin x = 0$ ✓

b) $4 + 4\sqrt{2}\cos x = 0$ b') $3 - 8\sin x = 1$ ✓

c) $\tan x = 1$

d) $3\sin x + 2 = 0$

e) $5\cos x - 3 = 0$

A' étudier sur $[0; 2\pi]$
 $f(x) = 5\cos x - 3$

f) $2\tan x - 1 = 3 - \tan x$

2) Résoudre les équations:

$$a) \quad 6 \cdot \sin^2 x - \sin x - 1 = 0$$

$$b) \quad 4 \cos^2 x - 5 \cos x - 6 = 0$$

$$b) \quad 4t^2 - 5t - 6 = 0$$

$$\Leftrightarrow t = \frac{5 \pm \sqrt{25 + 96}}{8} = \frac{5 \pm 11}{8} \begin{cases} \frac{16}{8} = 2 \\ -\frac{6}{8} = -\frac{3}{4} \end{cases}$$

$$\Rightarrow \cos x = 2$$

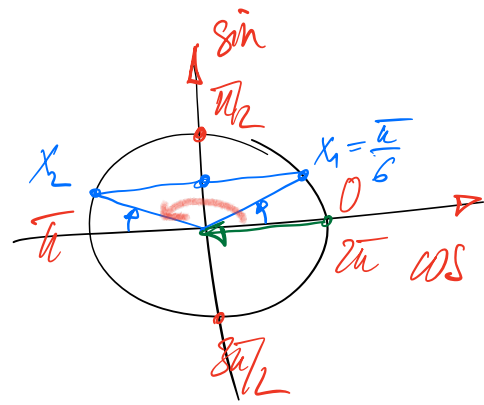
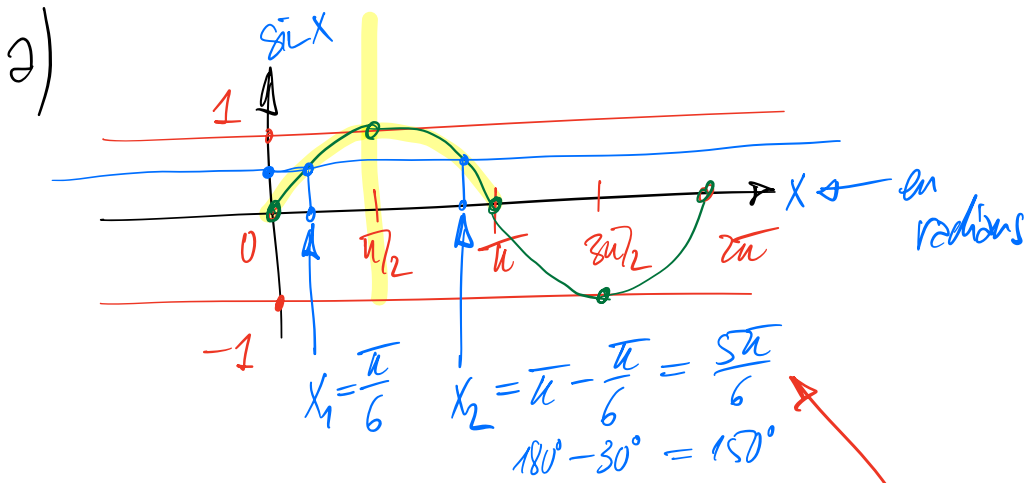
$$\cos x = -\frac{3}{4}$$

$$\cos^{-1}\left(-\frac{3}{4}\right) \approx 138,59038^\circ$$

$$\downarrow \\ \approx 2,419 \text{ rad}$$

$$x = \pm 2\pi \cos\left(-\frac{3}{4}\right) + k \cdot 2\pi$$

$$x \approx \pm 2,419 + k \cdot 2\pi$$



$$1 - 2\sin x = 0$$

$$1 = 2\sin x$$

$$2\sin x = 1 \quad \cdot \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$\boxed{\sin x = 0.5}$$

↓ blquette $\sin^{-1}$

$$x = 30^\circ$$

$$x = \frac{30^\circ \cdot \pi}{180^\circ} = \frac{\pi}{6} = x_1$$

$$\frac{5\pi}{6} = x_2$$

$$\frac{\alpha_{\text{rad}}}{\pi} = \frac{\alpha_{\text{deg}}}{180^\circ}$$

$$\frac{\alpha_{\text{rad}}}{\pi} = \frac{30^\circ}{180^\circ} \cdot \pi$$

$$X_1 = \left\{ \frac{\pi}{6}; \frac{\pi}{6} + 2\pi; \frac{\pi}{6} + 4\pi; \frac{\pi}{6} + 6\pi; \dots \right. \\ \left. \frac{\pi}{6} - 2\pi; \frac{\pi}{6} - 4\pi; \dots \right. \quad \alpha_{red} = \frac{30^\circ}{180^\circ} \cdot \pi$$

$$\alpha_{red} = \frac{1}{6} \cdot \pi = \frac{\pi}{6}$$

$$X_1 = \frac{\pi}{6} + k \cdot 2\pi \quad k \in \mathbb{Z}$$

$$X_2 = \frac{5\pi}{6} + k \cdot 2\pi \quad k \in \mathbb{Z}$$

$$2') \quad 1 - 3\sin X = 0$$

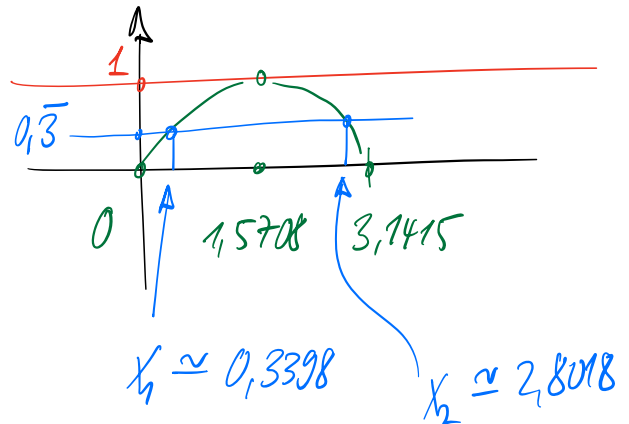
$$1 = 3\sin X$$

$$\sin X = \frac{1}{3}$$

$$\sin^{-1}$$

$$X \approx 19,4712^\circ$$

$$X \approx \frac{19,4712^\circ \cdot \pi}{180^\circ} \approx 0,3398$$



$$X_1 \approx 0,3398$$

$$X_1 \approx 0,3398 + k \cdot 6,2831$$

$$k = 0 \quad \begin{matrix} 1 & 2 & 3 \dots \\ -1 & -2 & -3 \dots \end{matrix}$$

$$X_2 \approx \pi - 0,3398 \approx 2,8018 \quad X_2 \approx$$

$$k \in \mathbb{Z}$$

b) Résoudre

$$3 - 8 \sin x = 1$$

$$3 - 1 = 8 \sin x$$

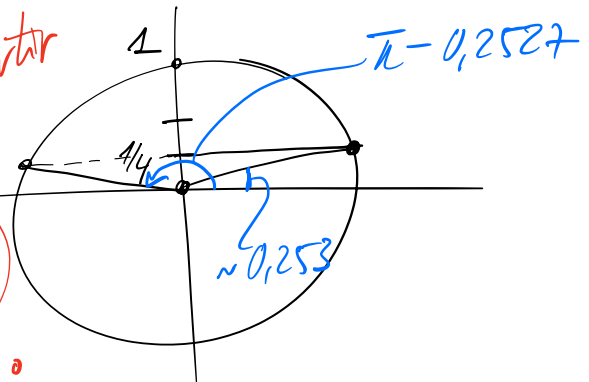
$$2 = 8 \sin x$$

$$\sin x = \frac{1}{4}$$

si degrés, convertir

$\sin^{-1}$
 $\sin$ $\pi/30$
 $\sim 14,47^\circ$

$$\alpha_{rad} = \frac{\alpha_{deg} \cdot \pi}{180^\circ}$$



$$\Rightarrow x = \arcsin\left(\frac{1}{4}\right) + k \cdot 2\pi$$

$$= 0,2527 + (k \cdot 2\pi)$$

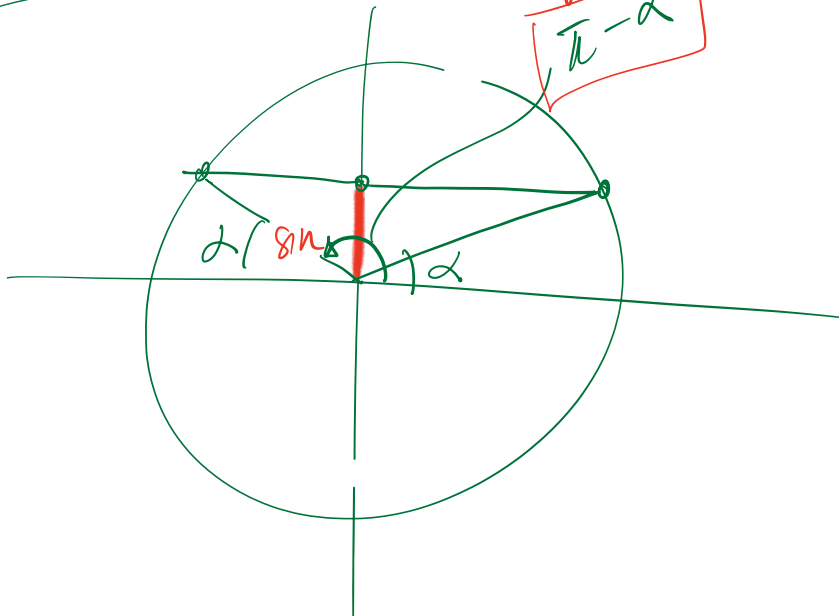
$$k = \begin{cases} 0 \\ 1, 2, 3, \dots \\ -1, -2, -3, \dots \end{cases}$$

$$x = \pi - 0,2527 + k \cdot 2\pi$$

$$= 2,8889 + k \cdot 2\pi$$

$$180^\circ$$

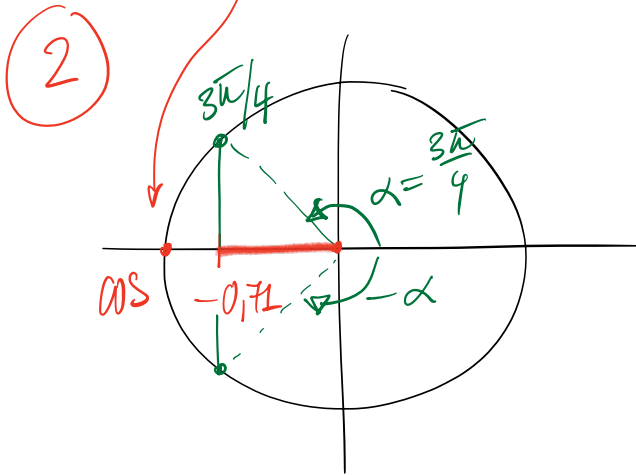
$$\pi - \alpha$$



$$b) \textcircled{1} 4 + 4\sqrt{2} \cdot \cos x = 0$$

$$4\sqrt{2} \cdot \cos x = -4$$

$$\cos x = \frac{-4}{4\sqrt{2}} = -\frac{1}{\sqrt{2}} \approx -0,707107$$



$$\textcircled{3} \cos^{-1}(-0,707107) = 135^\circ$$

$$\alpha = \frac{135^\circ}{180^\circ} \cdot \pi = \frac{2\pi}{36} \cdot \pi = \frac{3}{4}\pi$$

$$\textcircled{4} \alpha = -\frac{3\pi}{4}$$

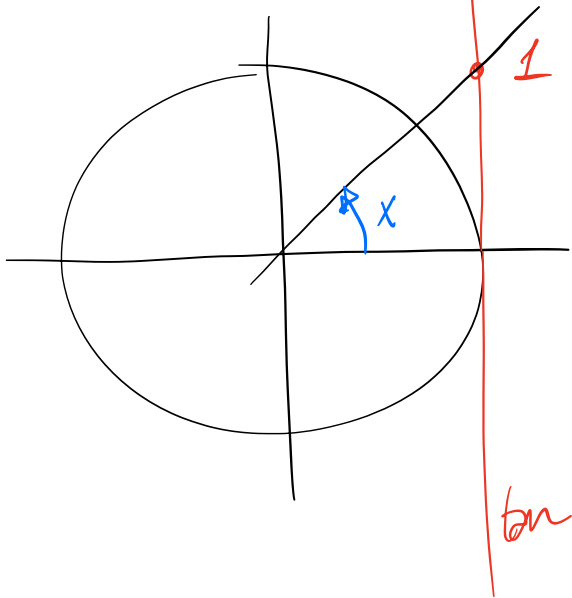
$$\textcircled{5} \begin{cases} x = \frac{3\pi}{4} + k \cdot 2\pi \\ x = -\frac{3\pi}{4} + k \cdot 2\pi \end{cases}$$

appartient à

$k \in \textcircled{\mathbb{Z}} = \{0; \pm 1; \pm 2; \pm 3; \dots\}$

k est un entier relatif

$$c) \tan x = 1$$



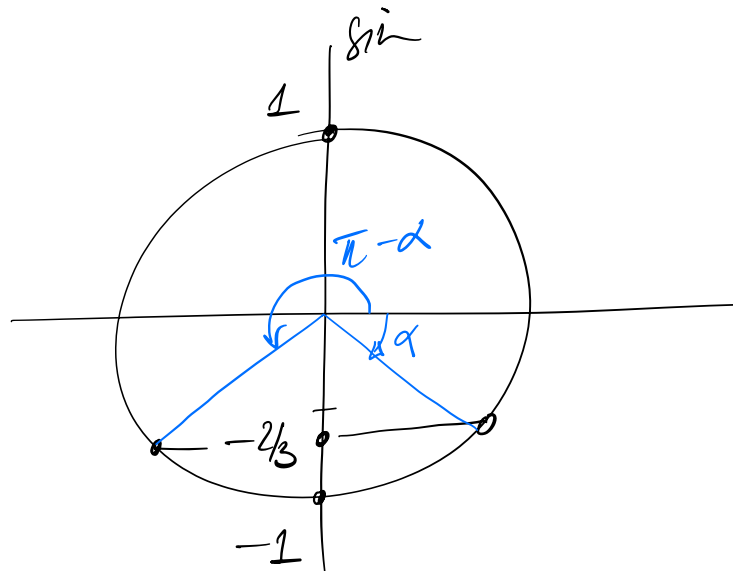
$$x = \arctan(1) = \frac{\pi}{4}$$

$$x = \frac{\pi}{4} + k \cdot \pi \quad k \in \mathbb{Z}$$

$$d) 3 \sin x + 2 = 0$$

$$3 \sin x = -2$$

$$\sin x = -\frac{2}{3}$$



$$\alpha = \arcsin\left(-\frac{2}{3}\right) \simeq -41,81^\circ \Rightarrow x \simeq -0,729728$$

$$x \simeq \boxed{-0,729728 + k \cdot 2\pi} \quad \leftarrow k \in \mathbb{Z} \rightarrow$$

$$x \simeq \pi - (-0,729728) + k \cdot 2\pi = \boxed{3,871320 + k \cdot 2\pi}$$

MARCHE A SUIVRE

- ① ISOLER $\cos x$, $\sin x$ ou $\tan x$
- ② DESSINER
- ③ CALCULER LA 1^{ÈRE} VALEUR $\left(\pi/30: \begin{matrix} \sin^{-1} \\ \cos^{-1} \\ \tan^{-1} \end{matrix} \right)$
- ④ DÉDUIRE LA 2^{ÈME} VALEUR ($\sin$ ET $\cos$)
- ⑤ ÉCRIRE LA OU LES FAMILLES DE SOLUTIONS.

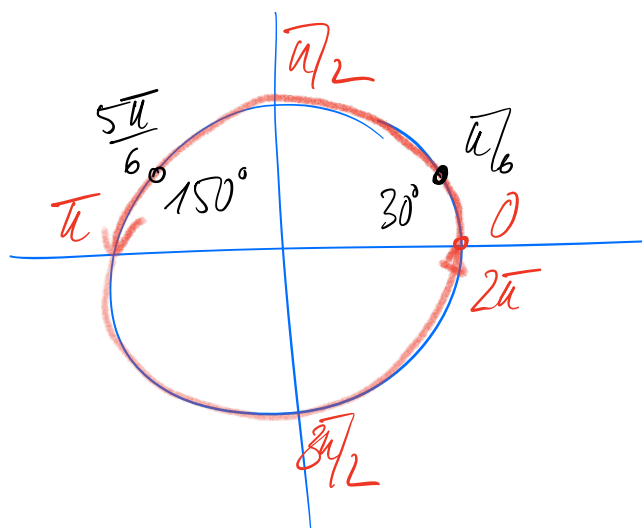
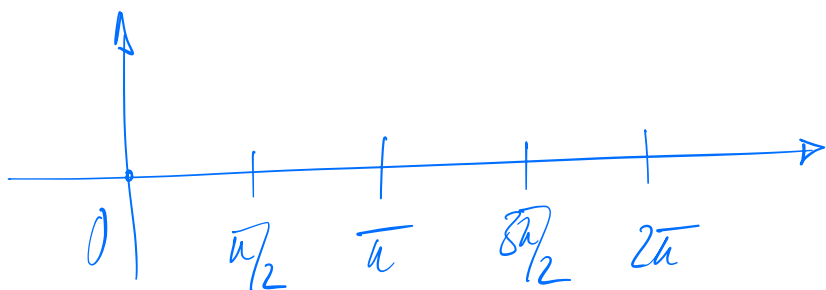
Etude de la fonction

$$f(x) = 1 - 2 \sin x$$

INTERVALLE D'ÉTUDE

$\sin, \cos : [0; 2\pi]$

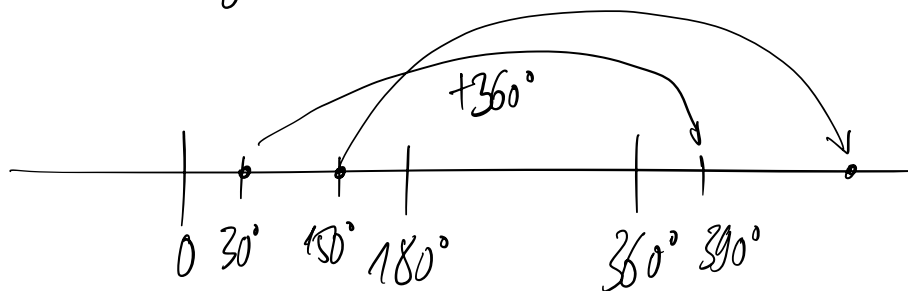
$\tan :]-\frac{\pi}{2}; \frac{\pi}{2}[$



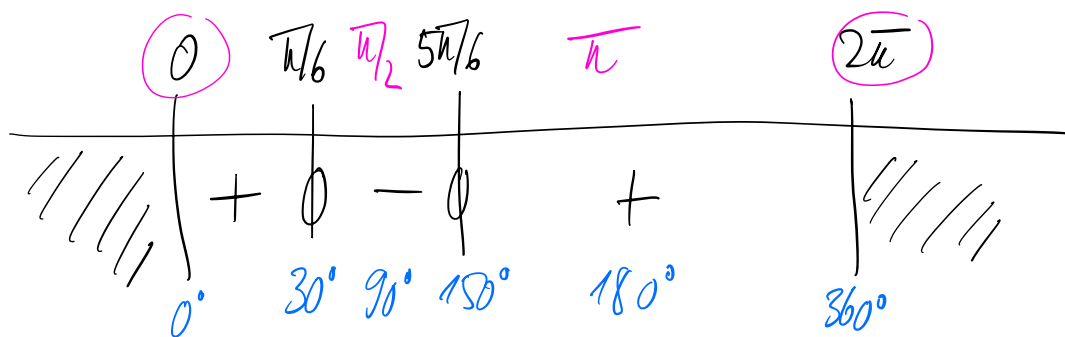
(1) $D_f = \mathbb{R}$ ($\sin, \cos$)

Zéros : $\frac{\pi}{6} + k \cdot 2\pi$ ($30^\circ + k \cdot 360^\circ$)

$\frac{5\pi}{6} + k \cdot 2\pi$ ($150^\circ + k \cdot 360^\circ$)



Signes:

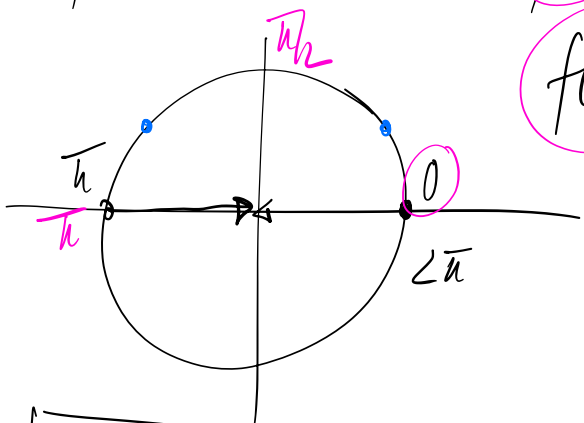


$$f(x) = 1 - 2\sin x$$

$$f(0) = 1 > 0$$

$$f(\pi) = 1 - 2 \cdot 0 = 1 > 0$$

$$f\left(\frac{\pi}{2}\right) = 1 - 2 \cdot 1 = -1$$



Dérivée et croissance:

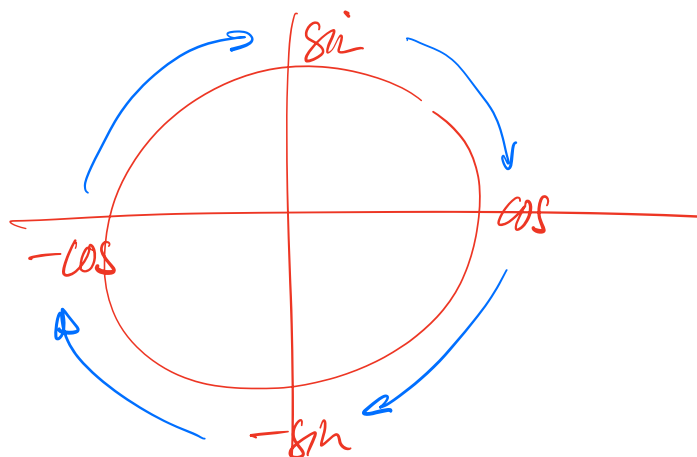
$$f(x) = 1 - 2\sin x$$

$$(\sin x)' = \cos x$$

$$f'(x) = (1)' - 2 \cdot (\sin x)'$$

$$= 2 \cdot -\cos x$$

$$= \boxed{-2\cos x}$$



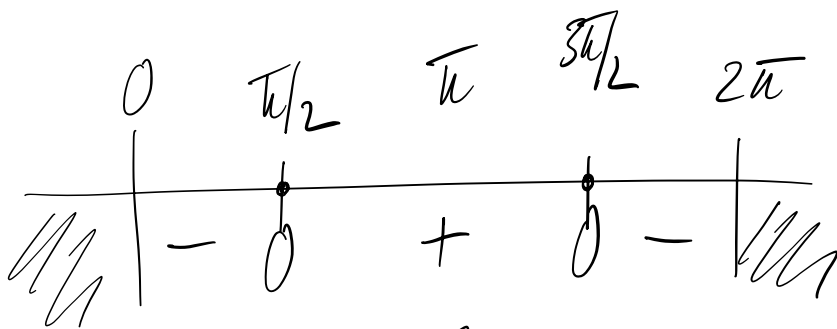
$$-2\cos x = 0$$

$$\Leftrightarrow \boxed{\cos x = 0}$$

$$x = \pm \frac{\pi}{2} + k2\pi$$

$$\text{Entre } 0 \text{ et } 2\pi: \frac{\pi}{2}; -\frac{\pi}{2} + 2\pi = \left(\frac{3\pi}{2}\right)$$

$$-2\cos x$$

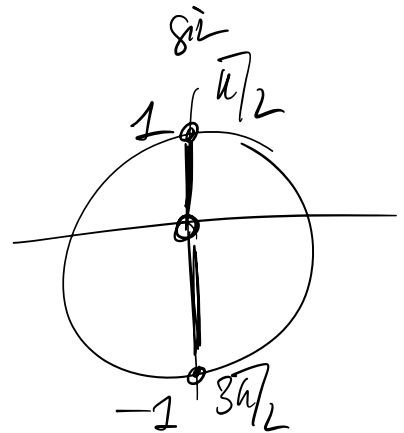


MIN
 $(\frac{\pi}{2}; -1)$

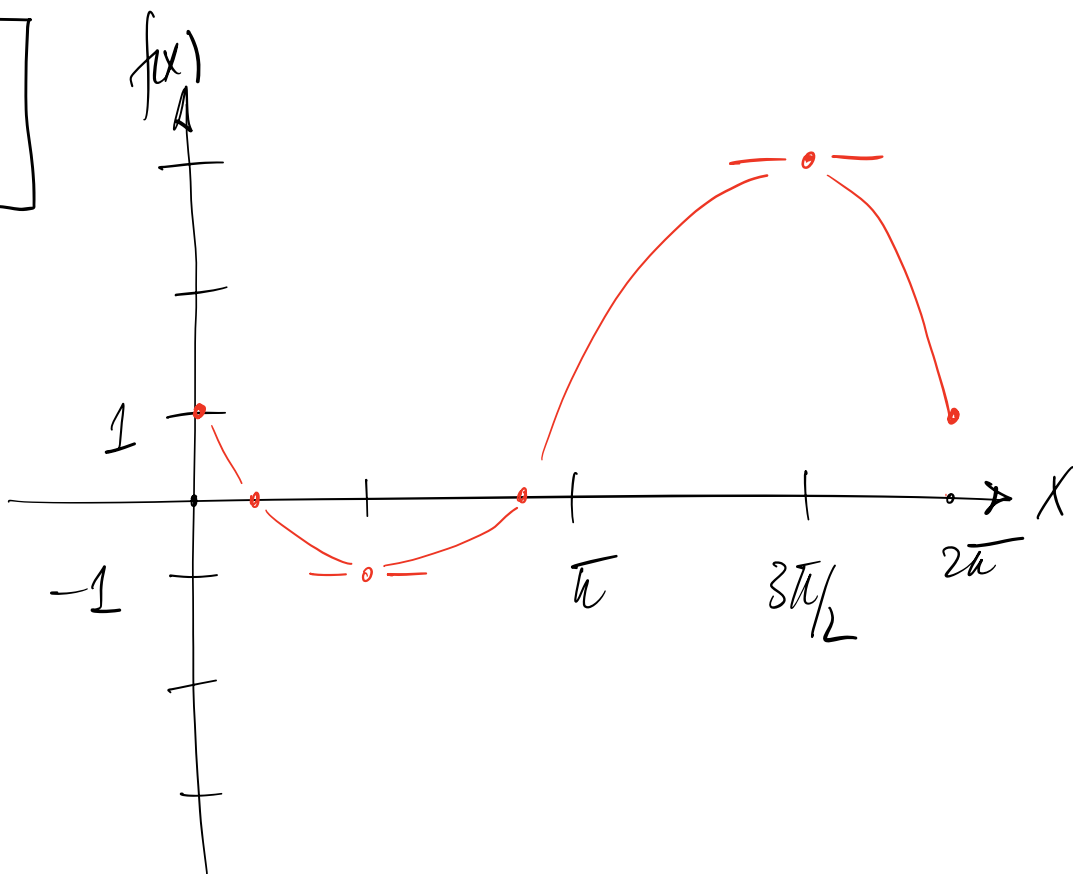
MAX
 $(\frac{3\pi}{2}; 3)$

$$f(x) = 1 - 2\sin^2 x$$

$$f(\frac{\pi}{2}) = -1 \quad f(\frac{3\pi}{2}) = 3$$



Graph



EXERCISE 2

$$2) \quad 6 \sin^2 x - \sin x - 1 = 0$$

$$\sin x = t$$

$$6t^2 - t - 1 = 0$$

$$t = \frac{1 \pm \sqrt{1 - 4 \cdot 6 \cdot (-1)}}{12} = \begin{cases} \frac{6}{12} = \frac{1}{2} \\ -\frac{4}{12} = -\frac{1}{3} \end{cases}$$

$$x = \arcsin \frac{1}{2} + k2\pi$$

$$x = \pi - \arcsin \frac{1}{2} + k2\pi$$

$$(1) \quad \sin x = \frac{1}{2}$$

$$x = \arcsin \left(-\frac{1}{3}\right) + k2\pi$$

$$x = \pi - \arcsin \left(-\frac{1}{3}\right) + k2\pi$$

$$(2) \quad \sin x = -\frac{1}{3}$$

$$e) \quad 5 \cos x - 3 = 0$$

$$\cos x = \frac{3}{5}$$

$$x \simeq 0,927295 + k \cdot 2\pi$$

$$x \simeq -0,927295 + k \cdot 2\pi$$

$$\alpha = \cos^{-1}\left(\frac{3}{5}\right) \simeq 53,13^\circ$$

$$x \simeq 0,927295$$

$$f) \quad 2 \tan x - 1 = 3 - \tan x$$

$$3 \tan x = 4$$

$$\tan x = \frac{4}{3}$$

$$x \approx 0,927295 + k \cdot \pi$$

$$\alpha = \arctan\left(\frac{4}{3}\right) \approx 53,13^\circ$$

$$x \approx 0,927295$$

Étude de $f(x) = 5\cos x - 3$

sur l'intervalle $[0; 2\pi]$

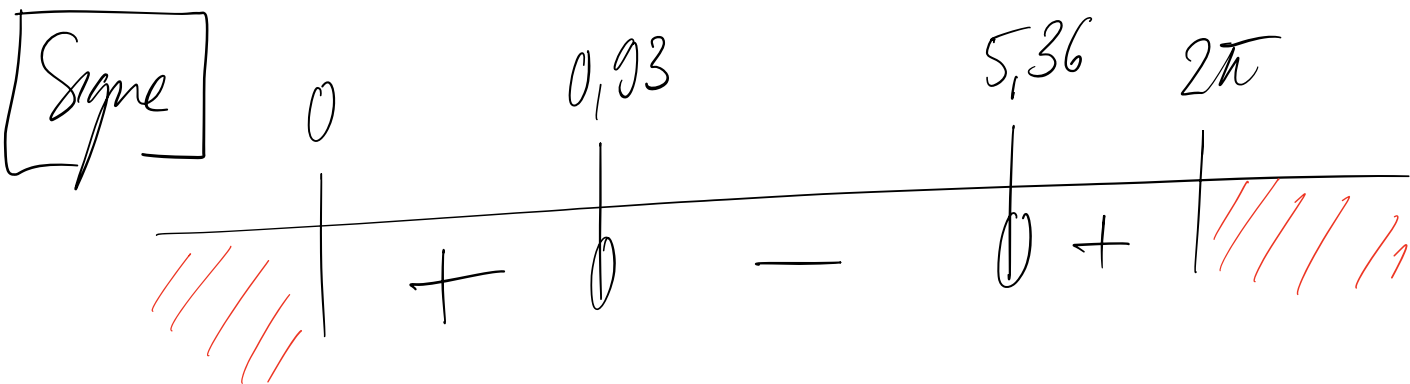
$$D_f = \mathbb{R}$$

Zéros (voir plus haut)

$$x \simeq 0,93 + k2\pi$$

$$x \simeq -0,93 + k2\pi \simeq 5,36 + k2\pi$$

$$0,93 \text{ et } 5,36 \in [0; 2\pi]$$



$$f(0) = 5\cos 0 - 3 = 5 \cdot 1 - 3 = 2 > 0$$

$$f(\pi) = 5\cos \pi - 3 = -5 - 3 = -8 < 0$$

$$f(2\pi) = 2 > 0$$

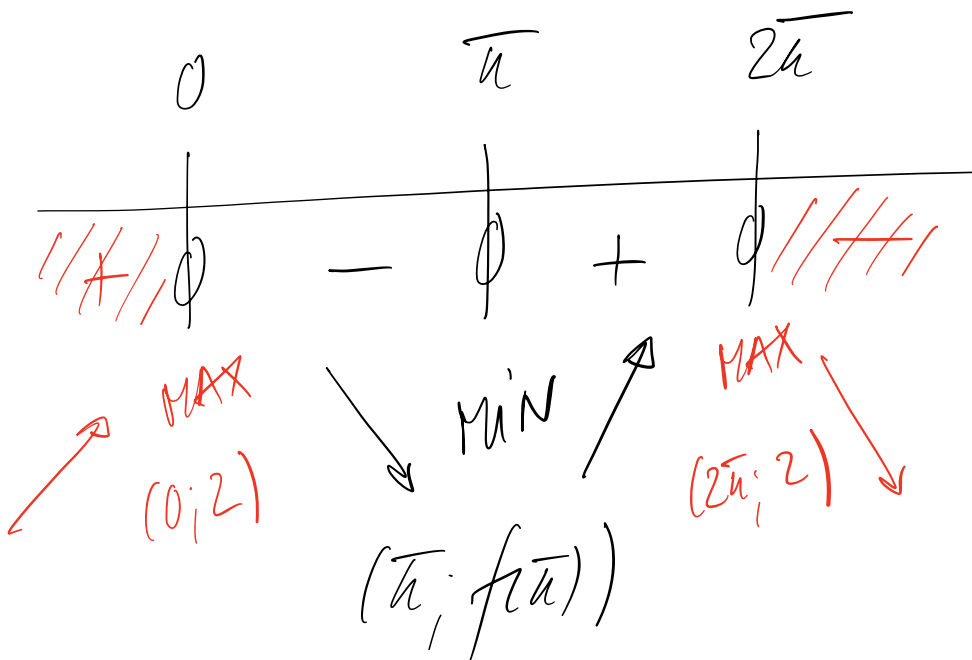
Dérivée et croissance

$$f'(x) = (5\cos x - 3)' = 5 \cdot (\cos x)' - 0$$

$$= 5 \cdot (-\sin x) = -5\sin x$$

$$f'(x) = 0 \Leftrightarrow \sin x = 0$$

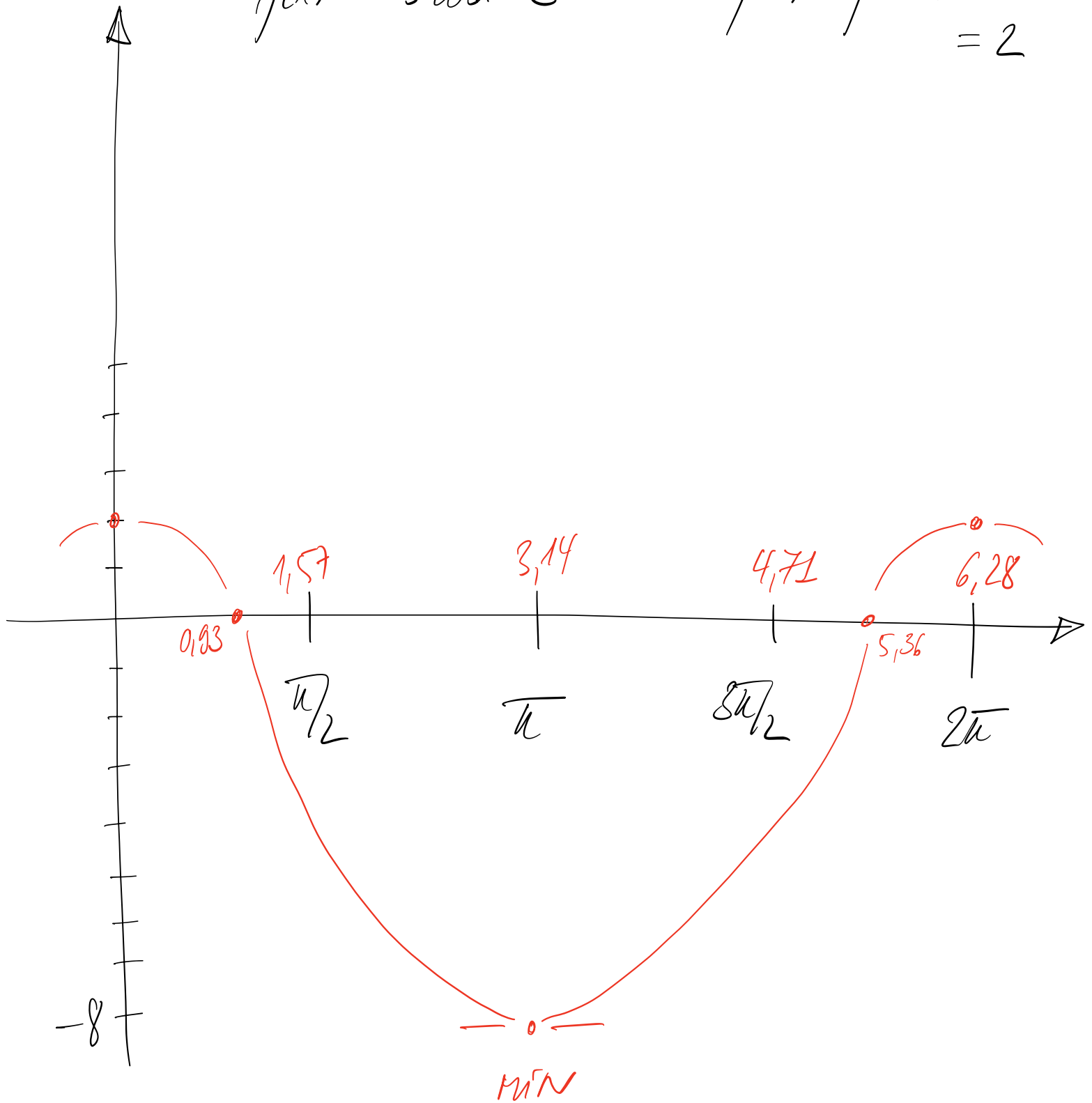
$$\Leftrightarrow x = k \cdot \pi$$



$$f(\pi) = -8$$

$$f(x) = 5\cos x - 3$$

$$f(0) = f(2\pi) = 5 - 3 = 2$$



Example: $4 \sin^2 x = 4 \sin x + 3$

$$\Leftrightarrow 4 \sin^2 x - 4 \sin x - 3 = 0$$

$$\sin x = t$$

$$4t^2 - 4t - 3 = 0$$

$$\Delta = (-4)^2 - 4 \cdot 4 \cdot (-3) = 16 - (-48) = 16 + 48 = 64$$

$$t = \frac{4 \pm \sqrt{16 + 48}}{8} = \frac{4 \pm \sqrt{64}}{8} = \frac{4 \pm 8}{8} = \begin{cases} 3/2 = 1.5 \\ -1/2 \end{cases}$$

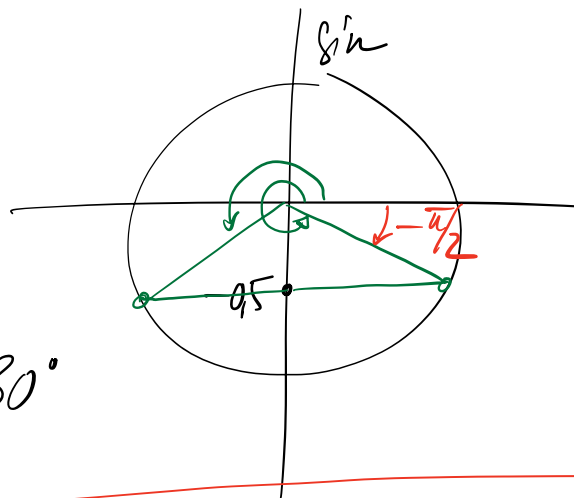
$$t = \sin x$$

$$\sin x = 1.5 > 1 \quad \downarrow$$

$$\sin x = -0.5$$

$$x = \arcsin(-0.5) = -30^\circ$$

$$x = -\frac{\pi}{6}$$



$$x = -\frac{\pi}{6} + k2\pi$$

$$x = \pi - \frac{\pi}{6} + k2\pi = \frac{5\pi}{6} + k \cdot 2\pi$$

EXERCICE 3

Répondre.

$$a) \quad 12 \sin^2 x + 7 \sin x = 12$$

$$b) \quad \tan^2 x = 1 - \tan x$$

$$a) \quad 12t^2 + 7t - 12 = 0$$

$$t = \frac{-7 \pm \sqrt{49 + 4 \cdot 144}}{24} = \frac{-7 \pm \sqrt{625}}{24} = \frac{-7 \pm 25}{24}$$

$$t = \begin{cases} 18/24 = 3/4 = 0,75 \\ -32/24 = -4/3 = -1,3 \end{cases}$$

$\sin x = -1,3$ est impossible

$$\sin x = 0,75$$

$$\arccos(0,75) \cong 48,5904^\circ \text{ ce}$$

qui donne $\approx 0,8481$ rad

$\Rightarrow$

$$x \approx 0,8481 + k \cdot 2\pi$$

$$x \approx \pi - 0,8481 + k \cdot 2\pi \approx 2,2935 + k \cdot 2\pi$$

$$b) \tan^2 x = 1 - \tan x$$

$$\Leftrightarrow \tan^2 x + \tan x - 1 = 0$$

$$\boxed{\tan x = t}$$

$$t^2 + t - 1 = 0 \Leftrightarrow t = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Leftrightarrow t \approx \begin{matrix} -1,618 \\ 0,618 \end{matrix}$$

$$\arctan(0,618) \approx 31,7175$$

$$\arctan(-1,618) \approx -58,2825$$

$$t = \tan x$$

$$\tan x = 0,618 \quad \Leftrightarrow$$

$$x \simeq 0,5536 + k \cdot \pi$$

$$\tan x = -1,618 \quad \Leftrightarrow$$

$$x \simeq -1,0172 + k \cdot \pi$$