TE OS du jeudi 12 décembre

11 abcde

12 abc

13 a CRM 4

14 a

15 ab

$\boxed{15\ b}$

$$\frac{1}{(k+1)^2-1} = \frac{1}{k^2+2k+1-1} = \frac{1}{k^2+2k} = u_k$$

$\sum \frac{1}{k^2}$ converge (Riemann)

$0 \leq u_k < v_k$ et que $\sum v_k$ converge, alors $\sum u_k$ converge aussi.

$k^2+2k > k^2$ si $k \geq 1$

$AB>0$ et $A>B \Rightarrow \frac{1}{A} < \frac{1}{B}$

$$\frac{1}{k^2+2k} < \frac{1}{k^2}$$

$$\Rightarrow \quad u_k = \frac{1}{(k+1)^2-1} = \frac{1}{k^2+2k} < \frac{1}{k^2} = v_k \quad \forall k \geq 1$$

Critère de comparaison

$\Rightarrow \sum u_k$ converge car $\sum v_k$ converge

$$\frac{\frac{1}{(k+1)!}}{\frac{1}{k!}} = \frac{1}{(k+1)!} \cdot \frac{k!}{1} =$$

$$= \frac{k!}{(k+1)!} = \frac{k!}{(k+1) \cdot k!} = \frac{1}{k+1} \xrightarrow{k \to \infty} 0$$

$$\frac{2^k}{2^{k+1}} = \frac{\cancel{2^k}}{2 \cdot \cancel{2^k}} = \frac{1}{2}$$

$$= 2^{k-(k+1)} = 2^{k-k-1} = 2^{-1}$$

or $\boxed{\frac{2^m}{2^n} = 2^{m-1}}$