$$\boxed{y,\ y',\ y'',\ y''',\ x,\ f(x),\ g(x),\ \text{—}}$$

y est fonction de x

$$\frac{3y}{x - y'} = \frac{1}{\sin(x)}$$

$$\boxed{y,\ y',\ x,\ f(x),\ \ldots}$$

<u>Exemple:</u> $\dfrac{1}{x y'} = \sin x$ On cherche y

$$\frac{1}{y'} = x \sin x$$

$$\frac{1}{x \sin x} = y' \qquad \frac{1}{x \sin x}\, dx = dy$$

$$\int \frac{1}{x \sin x}\, dx = y$$

$$\frac{y'}{x} = \sin(x)$$

$$y' = x \sin(x)$$

$$y = \int x \sin x \, dx \quad ✓$$

$$\frac{y'}{xy} = \cos x \iff \frac{dy}{y} = x \cos x$$

$$\iff \ln(y) = \int x \cos x \, dx$$

$$\iff \boxed{y = e^{\int x \cos x \, dx}}$$

$$\boxed{y' = \frac{x^2}{1-y^2}}$$

$$(1-y^2)\,dy = x^2 \quad \Leftrightarrow \quad \int (1-y^2)\,dy = \int x^2\,dx$$

$$\boxed{y - \frac{1}{3}y^3 = \frac{1}{3}x^3 + c}$$

$$y'(1-y^2) = y'(x)\,(1-y(x)^2)$$

$$= (1-y(x)^2)\cdot y'(x)$$

$$\int (1-y(x)^2)\cdot y'(x)\,dx =$$

$2\,y(x)$

$$\int y'(x)\,dx - \int y(x)^2\,y'(x)\,dx =$$

$$y(x) - \int y(x)^2\,y'(x)\,dx =$$

$f(x) = x^2$

$$y(x) - \frac{1}{3}y(x)^3 + C \qquad \int f(y(x)) \cdot y'(x)\, dx =$$

$$y - \frac{1}{3}y^3 \qquad\qquad F(y(x))$$

$$(1-y(x)^2)\, y'(x) = (1-y^2)\, y' = (1-y^2)\, dy$$

$$\int (1-y(x)^2) \cdot y'(x)\, dx = \int (1-y^2)\, dy$$

$$y'(x) = \frac{x^2}{1-y(x)^2}$$

<u>Inconnue</u> $y(x)$

$$(1-y(x)^2)\, y'(x) = x^2$$

$$\int y'(x)\,dx - \int [y(x)]^2 \cdot y'(x) \cdot dx = \int x^2 dx$$

$( )'$

$$y(x) - \frac{1}{3}(y(x))^3$$

$$\int \sin(x^2) \cdot x \cdot dx =$$

$$\frac{1}{2}\int \sin(x^2) \cdot 2x \cdot dx =$$

$( )'$

$$\frac{1}{2}\left[-\cos(x^2)\right]$$

$$\int e^{y(x)}\, y'(x)\, dx = e^{y(x)} + c$$

$$\boxed{\int f(y(x)) \cdot y'(x)\, dx = F(y(x)) + c}$$

$$f(y(x)) \cdot y'(x) = g(x)$$

$$\int f(y(x))\, y'(x)\, dx = \int g(x)\, dx$$

$$F(y(x)) = G(x) + C$$

$$f(y)\, dy = g(x)$$

$$\int f(y)\, dy = \int g(x)\, dx$$

$$F' = f$$

$$G' = g$$

Exemple 2 p. 28

$$2y + (xy + 3x)y' = 0$$

Séparable ?

$$2y = -x(y+3)\,y'$$

$$-\frac{1}{x} = \frac{y+3}{2y}\,y'$$

$$\frac{y+3}{2y} = \frac{y}{2y} + \frac{3}{2y}$$

$$-\int \frac{1}{x}\,dx = \frac{1}{2}\int 1\,dy + \frac{3}{2}\int \frac{1}{y}\,dy$$

$$-\ln|x| + C = \boxed{\frac{1}{2}y + \frac{3}{2}\ln|y|}$$

$$\Leftrightarrow\ -\frac{1}{2}y - \ln|x| + c = \frac{3}{2}\ln|y|$$

$$\Leftrightarrow\ -\frac{1}{3}y - \frac{2}{3}\ln|x| + c = \ln|y|$$

$$\Leftrightarrow \quad y = e^{\left(-\frac{1}{3}y - \frac{2}{3}\ln|x| + c\right)}$$

$$\Leftrightarrow \quad y = e^{-\frac{1}{3}y} \cdot e^{-\frac{2}{3}\ln|x|} \cdot e^{c}$$

$$y' = \frac{y\cos x}{1+2y^2}$$

$$\Leftrightarrow \int \frac{1+2y^2}{y}\,dy = \int \cos x\,dx$$

$$\Leftrightarrow \int \frac{1}{y}\,dy + 2\int y\,dy = \int \cos x\,dx$$

$$\Leftrightarrow \boxed{\ln|y| + y^2 = \sin x + C}$$

$y(0) = 1 \qquad x = 0,\ y = 1$

$$\ln|y(0)| + (y(0))^2 = \sin(0) + C$$

$$\ln|1| + 1^2 = 0 + C \quad \Rightarrow \quad C = 1$$

$\boxed{2.1}$

a) $$\frac{dy}{\cos x} - 2y\,dx = 0$$

$$\frac{dy}{\cos x} = 2y\,dx$$

$$\frac{dy}{2y} = \cos x\,dx$$

$$\frac{1}{2} \cdot \frac{1}{y} \cdot dy = \cos x \cdot dx$$

$$\frac{1}{2} \ln|y| = \sin x + C$$

$$\ln|y| = 2\sin x + 2C$$

$$|y| = e^{(2\sin x + 2C)}$$

$$e^{m+n} = e^m \cdot e^n$$

$$|y| = e^{2\sin x} \cdot e^{2C}$$

$$y = \underbrace{\pm e^{2C}}_{K} \cdot e^{2\sin x}$$

$$\boxed{y = K \cdot e^{2\sin x}} \quad K \in \mathbb{R}$$

famille de solutions

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

Noyau

Résoudre le système $H \cdot x = 0$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 3 & -1 \\ 0 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1/3 \\ 0 & 1 & 1/4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 7/12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 1 \end{pmatrix}$$

$x_1 - 2x_2 + x_3 = 0 \Rightarrow x_1 = 0$

$x_2 - 1/3 x_3 = 0 \Rightarrow x_2 = 0$

$x_3 = 0$

$\Rightarrow$ L'unique solution de $H \cdot X = 0$ est $X = (0;0;0)$

$(x_1;x_2;x_3) = (0;0;0)$

$\Rightarrow$ $\ker H = 0 = \{(0;0;0)\}$

$\Rightarrow$ $\operatorname{Im} H = \mathbb{R}^3$

H est inversible $\Rightarrow$ h bijective

injective ✓

surjective ✓

$$\ln|y| + c_1 = \ln|x| + c_2 \Leftrightarrow \ln|y| = \ln|x| + \underbrace{c_2 - c_1}_{c}$$

$$e^{\ln|y| + c_1} = e^{\ln|x| + c_2}$$

$$e^{\ln|y|} \cdot e^{c_1} = e^{\ln|x|} \cdot e^{c_2}$$

$$|y| \cdot e^{c_1} = |x| \cdot e^{c_2}$$

$$\left.\begin{array}{l} e^{c_1} = k_1 \\ e^{c_2} = k_2 \end{array}\right\} \neq 0$$

$$|y| = \frac{k_2}{k_1} \cdot |x|$$

avec $k_1, k_2 > 0$

$$y = \pm \frac{k_2}{k_1} \cdot x$$

$k \in \mathbb{R}$

$$\boxed{\ln(xy) = \ln(x) + \ln(y)}$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(y^{-1}) = -1 \cdot \ln(y)$$

$$\boxed{\ln(x^q) = q \cdot \ln(x)}$$

$$\ln(xy^{-1}) = \ln(x) + \ln(y^{-1})$$

$$= \ln(x) - \ln(y)$$

$$\begin{pmatrix} 7 & -4 & 8 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{cases} 7x_1 = 4x_2 - 8x_3 \\ 3x_2 = -x_3 \\ x_3 = k \end{cases}$$

1 variable libre $\longrightarrow$ $\dim(\ker G) = 1$

$x_2 = -\frac{1}{3} \cdot k$ / $7x_1 = -\frac{4}{3}k - 8k$

$$x_1 = \left(-\frac{4}{21} - \frac{8}{7}\right) \cdot k = \left(\frac{-28}{21}\right) \cdot k = -\frac{4}{3}k$$

Finalement:

$$\begin{cases} x_1 = -\frac{4}{3}k \\ x_2 = -\frac{1}{3}k \\ x_3 = k \end{cases}$$

$\ker G : \left\{ k \cdot \begin{pmatrix} -4/3 \\ -1/3 \\ 1 \end{pmatrix} \mid k \in \mathbb{R} \right\}$

C'est la droite dont le vecteur directeur est $\begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$