Vecteur : Élément d'un espace vectoriel U (ensemble) sur un corps $\mathbb{R}$ ou $\mathbb{C}$ ou $\mathbb{F}_p = \{0; \dots; p-1\}$

nombres

$U = \{ax^2+bx+c \mid a,b,c \in \mathbb{R}\}$ polynômes ($\deg \leq 2$) sur $\mathbb{R}$

$+$ ✓

$k(ax^2+bx+c) = akx^2+bkx+ck \quad k \in \mathbb{R}$

$$U \xrightarrow{\sim} \mathbb{R}^3$$

$$ax^2+bx+c \longmapsto \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$U \xrightarrow{\frac{d}{dx}} U$ endomorphisme « dérivée »

$$ax^2+bx+c \longmapsto 2ax+b = 0\cdot x^2 + 2a\cdot x + b$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \longmapsto \begin{pmatrix} 0 \\ 2a \\ b \end{pmatrix} = \begin{pmatrix} 0\cdot a + 0\cdot b + 0\cdot c \\ 2\cdot a + 0\cdot b + 0\cdot c \\ 0\cdot a + 1\cdot b + 0\cdot c \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+2y+3z \\ 4x+2y+z \\ x+y+z \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto$$

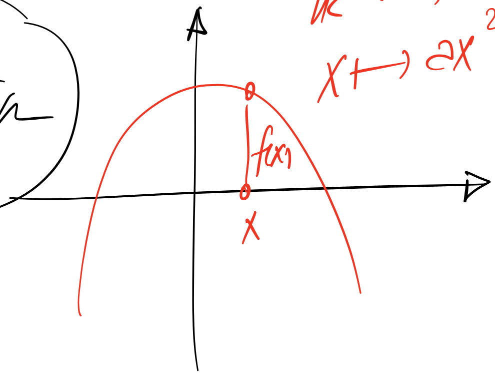
Base canonique de $\mathbb{R}^3$:

$$\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$ax^2+bx+c \longmapsto \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$\boxed{(X^2, X, 1) \text{ Base canonique de } U}$

Fonction



$$\mathbb{R} \to \mathbb{R}$$
$$x \mapsto ax^2+bx+c$$

$x \in \mathbb{R}$

Polynôme

Vecteur

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Objet algébrique

$$a\xi^2+b\xi+c$$

symbole

(calcul symbolique)

$$P_2 = \{a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in \mathbb{R}\} \xrightarrow{\sim} \left\{\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} \middle| a_i \in \mathbb{R}\right\}$$ 1.3.24

$(1, x, x^2)$

$$\boxed{1 \mapsto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad x \mapsto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad x^2 \mapsto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$$

$$\underbrace{\qquad\qquad\qquad}_{\mathbb{R}^3}$$

$$d(p) = (x+1) \cdot p'$$

$$d(\boxed{a_0 + a_1 x + a_2 x^2}) = (x+1) \cdot (a_0 + a_1 x + a_2 x^2)' = (x+1)(a_1 + 2a_2 x)$$

$$= a_1 x + 2a_2 x^2 + a_1 + 2a_2 x$$

$$= \boxed{a_1 + (a_1 + 2a_2) x + 2a_2 x^2}$$

linéaire

$$d\left(\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}\right) = \begin{pmatrix} a_1 \\ a_1 + 2a_2 \\ 2a_2 \end{pmatrix}$$

$$d\left(\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}\right) = \begin{pmatrix} 0 \cdot a_0 + 1 \cdot a_1 + 0 \cdot a_2 \\ 0 \cdot a_0 + 1 \cdot a_1 + 2 \cdot a_2 \\ 0 \cdot a_0 + 0 \cdot a_1 + 2 \cdot a_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$\Rightarrow$ d est un endomorphisme

$$f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = M \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow f \text{ est linéaire}$$

M matrice 3×3

$\boxed{1.5.13}$

$\lambda = 2 \qquad E_2 = \left\langle \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\rangle$

$\lambda = 3 \qquad E_3 = \left\langle \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\rangle$

$\mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "H"] \arrow[d, shift right] & \mathbb{R}^2 \arrow[d, shift left, "P^{-1}"] \\ \mathbb{R}^2 \arrow[r, "H'"'] \arrow[u, shift right, "P"'] & \mathbb{R}^2 \arrow[u, shift left] \end{tikzcd}$$

$\mathcal{B}' = \left(\begin{pmatrix} 2 \\ -1 \end{pmatrix}; \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right)$

$$P = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$$

$$H = P H' P^{-1}$$

$$H' = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x_1 = x_1$$
$$x_2 = 0$$
$$x_3 = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$