PRODUIT MATRICIEL

Tableau de nombres rect.

$$\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$$

$$\begin{cases} 3x_1 - 2x_2 = 1 \\ 2x_1 + 3x_2 = 2 \end{cases}$$

Forme:

$$\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ ← matricielle

$$x_1 \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} + x_2 \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ ← vectorielle

$$\begin{pmatrix} 3x_1 \\ 2x_1 \end{pmatrix} + \begin{pmatrix} -2x_2 \\ 3x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3x_1 - 2x_2 \\ 2x_1 + 3x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ ← usuelle

$$(x_1 \; x_2) \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 - 2x_2 \\ 2x_1 + 3x_2 \end{pmatrix}$$

$2 \times 2 \qquad 2 \times 1 \qquad 2 \times 1$

$1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 22 & 28 \\ 49 & 64 \end{pmatrix}$$

L1, L2 — C1, C1 — 11, 21

$2 \times 3 \qquad 3 \times 2$

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \checkmark = \begin{pmatrix} 9 & 12 & 15 \\ 7 & 11 & 15 \end{pmatrix}$$

$2 \times 2 \qquad 2 \times 3 \qquad 2 \times 3$

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$2 \times 2 \qquad 3 \times 2$

Pas définie

$$f(x) = \frac{x-2}{x^3+x^2+4x+4}$$

← A' décomposer en éléments simples

But: Calculer $\int f(x)\,dx$

$$f(x) = \frac{x-2}{x^2(x+1)+4(x+1)} = \frac{x-2}{(x^2+4)(x+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2+4}$$

x^2+4 : IRRÉDUCTIBLE DANS $\mathbb{R}[x]$

$$x^2+4 = (x+2i)(x-2i)$$

$$\Rightarrow f(x) = \frac{x-2}{(x+1)(x+2i)(x-2i)} = \frac{a}{x+1} + \underbrace{\frac{b}{x+2i} + \frac{c}{x-2i}}_{(*)}$$

$a \in \mathbb{R}$ $\quad b, c \in \mathbb{C}$

Mult. par $x+1$

$$\frac{x-2}{x^2+4 = (x+2i)(x-2i)} = a + (x+1)\cdot(*)$$

$$x = -1$$

$$\frac{-3}{1+4} = a + 0\cdot(\#)$$

$$a = \frac{-3}{5}$$

Mult. par $x+2i$

$$\frac{x-2}{(x+1)(x-2i)} = \frac{a\cdot\underbrace{(x+2i)}_{-2i \to 0}}{x+1} + \frac{b\,\cancel{(x+2i)}}{\cancel{(x+2i)}} + \frac{c\cdot\underbrace{(x+2i)}_{-2i \to 0}}{(x-2i)}$$

$$x = -2i$$

$$\frac{-2i-2}{(-2i+1)(-4i)} = 0 + b + 0$$

$$b = \frac{-2i-2}{-8-4i} = \frac{-2(i+1)}{-4(2+i)} = \frac{1}{2}\cdot\frac{(i+1)(2-i)}{5}$$

$$= \frac{1}{2}\cdot\frac{2i+1+2-i}{5} = \frac{1}{2}\,\frac{i+3}{5}$$

Mult. por $x-2i$

$$\frac{x-2}{(x+1)(x+2i)} = c + (x-2i)\cdot(**)$$

$x=2i$ $\quad \dfrac{2i-2}{(2i+1)\,4i} = c$

$$\frac{2(i-1)}{-8+4i} = c$$

$$\frac{1}{2}\cdot\frac{i-1}{i-2} = \frac{(i-1)(i+2)}{2(-1-4)}$$

$$= -\frac{1}{10}\cdot(-1+i-2)$$

$$= -\frac{1}{10}\cdot(i-3)$$

$$\Rightarrow f(x) = -\frac{3}{5}\cdot\frac{1}{x+1} + \frac{1}{10}\,\frac{(2i+3)}{x+2i} - \frac{1}{10}\,\frac{(2i-3)}{x-2i}$$

$$= -\frac{3}{5}\cdot\frac{1}{x+1} + \frac{1}{10}\,\frac{(2i+3)(x-2i) - (2i-3)(x+2i)}{x^2+4}$$

$$= -\frac{3}{5}\,\frac{1}{x+1} + \frac{1}{10}\,\frac{\cancel{xi}+2+3x-\cancel{6i}-(\cancel{xi}-2-3x-\cancel{6i})}{x^2+4}$$

$$= -\frac{3}{5}\,\frac{1}{x+1} + \frac{1}{10}\cdot\frac{6x+4}{x^2+4}$$

$$= -\frac{3}{5}\,\frac{1}{x+1} + \frac{1}{5}\cdot\frac{3x+2}{x^2+4}$$

$$\left(\exp\left(\left(\frac{1+x^2}{1-x^2}\right)^{1/2}\right)\right)' = \exp'\left(\left(\frac{1+x^2}{1-x^2}\right)^{1/2}\right) \cdot \left(\left(\frac{1+x^2}{1-x^2}\right)^{1/2}\right)'$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$= \exp\left(\left(\frac{1+x^2}{1-x^2}\right)^{1/2}\right) \cdot \frac{1}{2} \cdot \left(\frac{1+x^2}{1-x^2}\right)^{-\frac{1}{2}} \cdot \left(\frac{1+x^2}{1-x^2}\right)'$$

$$\left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2}$$