$f(x) = \ln(x)$ $\qquad n = 3 \quad a = 1 \qquad [1/2 ; 3/2]$

$f'(x) = \frac{1}{x}$

$f''(x) = -\frac{1}{x^2}$

$f'''(x) = \frac{2}{x^3} = 2 \cdot x^{-3}$ $\qquad (2 \cdot x^{-3}) = 2 \cdot (-3)\, x^{-4}$

$f^{IV}(x) = \frac{-6}{x^4}$

$$\left(\frac{2}{x^3}\right)' = \frac{2' \cdot x^3 - 2 \cdot (x^3)'}{x^6} = \frac{0 - 2 \cdot 3 \cdot x^2}{x^6} = \frac{-6 \cdot \cancel{x^2}}{x^4 \cdot \cancel{x^2}} = \frac{-6}{x^4}$$

$$\ln(x) = \ln(1) + \frac{1}{1} \cdot (x-1) - \frac{1}{1^2} \cdot (x-1)^2 \cdot \frac{1}{2} + \frac{2}{1^3} \cdot (x-1)^3 \cdot \frac{1}{3!} + R_3(x)$$

$$R_3(x) = \frac{(x-1)^4}{4!} \cdot \frac{-6}{c^4}$$

pour $c \in [1/2 ; 3/2]$

$$\ln(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + R_3(x)$$

$$|R_3(x)| \leq \frac{|(x-1)^4|}{4!} \cdot \underbrace{\sup_{t \in [0{,}5;1{,}5]} \left| \frac{-6}{t^4} \right|}$$

$$\frac{6}{0{,}5^4} = 6 \cdot 2^4 = 6 \cdot 16 = 96$$

$$\Rightarrow |R_3(x)| \leq (x-1)^4 \cdot \frac{96}{24} = (x-1)^4 \cdot 4 \leq \underbrace{0{,}5^4 \cdot 4}_{} = \left(\frac{1}{2}\right)^4 \cdot 2^2$$

sur $[0{,}5; 1{,}5]$

$$\leq 0{,}25$$

car $(x-1) \in [-0{,}5; 0{,}5]$

$\Rightarrow (x-1)^4$ est max

pour $(x-1) = 0{,}5$

$$\sum \frac{1}{(2k-1)\ x^k}$$

pô une série entière

$$\sum a_k \cdot x^k$$

changement de variable

$$\left(\frac{1}{x}\right)^k = y^k \Leftrightarrow \frac{1}{x} = y$$

$$\sum \frac{1}{(2k-1)} \cdot y^k$$