

Equations et fonctions trigonométriques

Il est demandé d'écrire les détails de tous les calculs.

Problème 1

Résoudre l'équation

$$2 \cos(x) + 2 - 3 \cos(x) = \cos(x) + 1$$

Problème 2

Résoudre l'équation

$$\cos^2(x) = 2 \cos(x) - 2 \sin^2(x)$$

Problème 3

Étudier la fonction donnée par

$$f(x) = 3 \cos(x) - 2$$

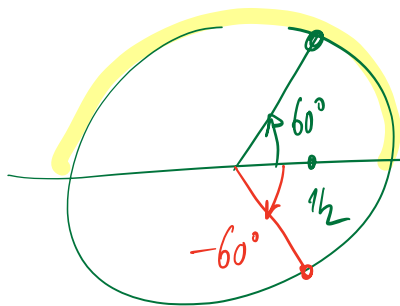
sur l'intervalle $[-\pi/2; 3\pi/2]$.

P1

$$-\cos(x) + 2 = \cos(x) + 1$$

$$1 = 2 \cos(x)$$

$$\cos(x) = \frac{1}{2} \left(\in [-1; 1] \right) \quad \boxed{\text{2nd}} \quad \boxed{\cos^{-1}} \quad 0,5$$



$$x = 60^\circ + k \cdot 360^\circ = \frac{\pi}{3} + k2\pi$$

$$k \in \mathbb{Z} = \{0; 1; -1; 2; -2; 3; -3; \dots\}$$

$$x = -60^\circ + k \cdot 360^\circ = -\frac{\pi}{3} + k2\pi$$

k ENTIER
RELATIF

$$\alpha_{\text{rad}} = \frac{1}{3} \cdot \pi = \frac{\pi}{3}$$

$$\frac{\alpha_{\text{rad}}}{\pi} = \frac{\alpha_{\text{deg}}}{180^\circ} \quad \frac{\alpha_{\text{rad}}}{\pi} = \frac{60^\circ}{180^\circ} = \frac{1}{3}$$

P2

$$\cos^2 x = 2 \cos x - 2 \sin^2 x$$

$$\cos^2 x + 2 \sin^2 x - 2 \cos x = 0$$

formule: $\boxed{\cos^2 x + \sin^2 x = 1} \Rightarrow \sin^2 x = 1 - \cos^2 x$

$$\cos^2 x + 2(1 - \cos^2 x) - 2 \cos x = 0$$

$$\cos^2 x + 2 - 2 \cos^2 x - 2 \cos x = 0$$

$$-\cos^2 x - 2 \cos x + 2 = 0 \quad \cdot (-1)$$

$$\cos^2 x + 2 \cos x - 2 = 0$$

$$(\cos x)^2 + 2 \cos x - 2 = 0$$

$\boxed{\cos x = y}$ changement de variable

$$y^2 + 2y - 2 = 0$$

$$y = \frac{-2 \pm \sqrt{4 + 4 \cdot 2}}{2} = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}$$

$\approx < \cancel{-2.73} \quad -2.73 \notin [-1; 1]$
 $\quad \quad \quad 0.73$

$$y = \cos x$$

$$\cos x \approx 0,73 \leftarrow \text{cf. P1} \quad \downarrow \quad \arccos(0,73) = 0,75 \text{ rad}$$

$$x \approx 0,75 \text{ rad}$$

$$\Rightarrow \boxed{\begin{aligned} x &\approx 0,75 + k \cdot 2\pi \\ x &\approx -0,75 + k \cdot 2\pi \end{aligned}} \leftarrow \text{cf. formule}$$

$$[P3] \quad f(x) = 3 \cos x - 2$$

$$\text{sur } \left[-\frac{\pi}{2}; \frac{3\pi}{2}\right] = I$$

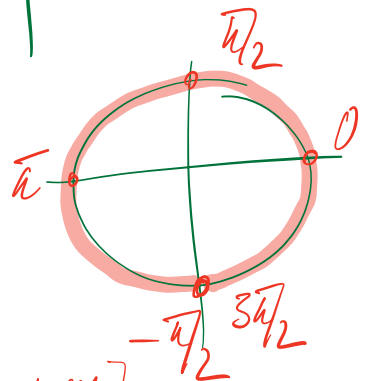
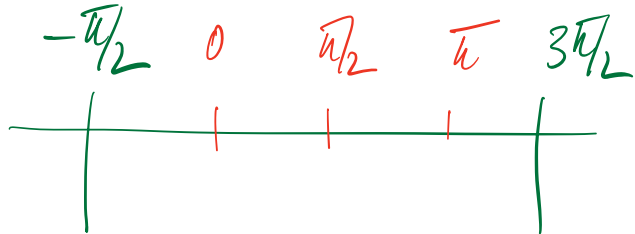
$$① \quad \mathcal{D}_f = I$$

Zeros: $3 \cos x - 2 = 0$

$$\cos x = \frac{2}{3} \approx 0,66$$

$$x \approx \pm 0,84 + k \cdot \underbrace{6,28}_{2\pi}$$

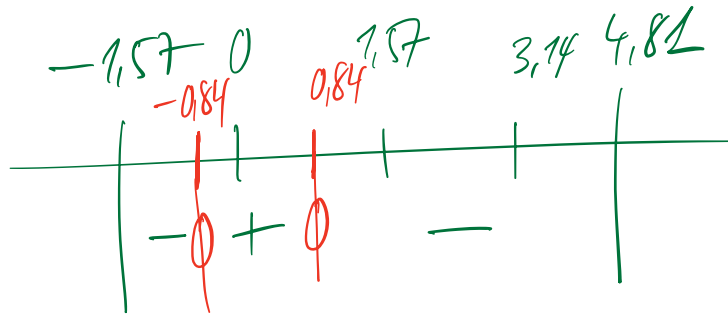
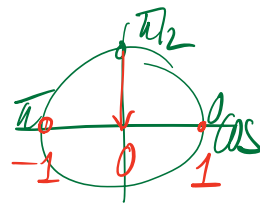
$\swarrow$ red



$$\triangle \text{ Contrainte : } x \in \left[-\frac{\pi}{2}; \frac{3\pi}{2}\right] = [-1,57; 4,81]$$

$$x = 0,84 \in [-1,57; 4,81]$$

$$x = -0,84 \in [-1,57; 4,81]$$



-2

$$3 \cos x - 2$$

$$3 \cos\left(-\frac{\pi}{2}\right) - 2 = -2$$

$$3 \cos(0) - 2 = 1$$

$$3 \cos(\pi) - 2 = -3 - 2$$

② Dérivée et croissance :

$$f(x) = 3 \cos x - 2$$

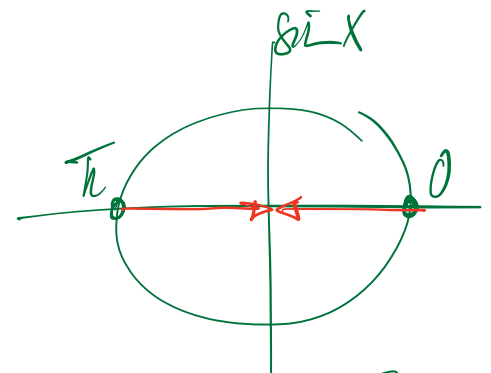
$$f'(x) = 3 \cdot (-\sin x) = -3 \sin x$$

$$f'(x) = 0 \Leftrightarrow \sin x = 0$$

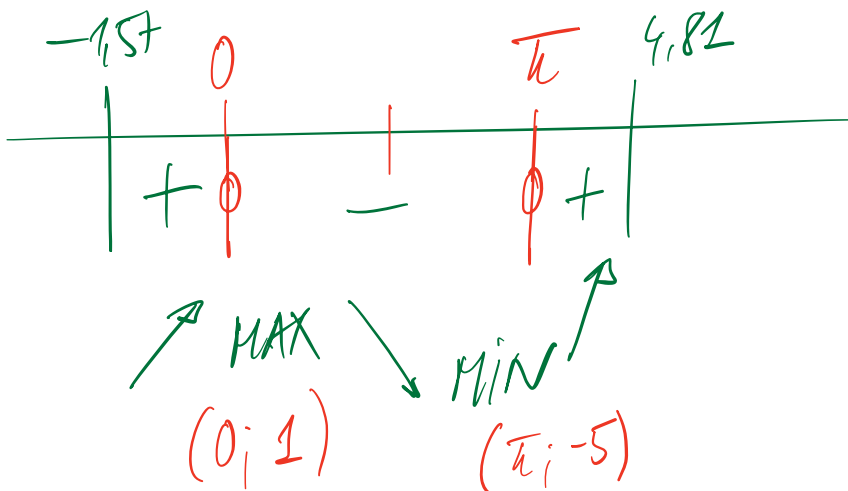
$$x = \boxed{0} + k2\pi$$

$$x = \boxed{\pi - 0} + k2\pi = \pi + k2\pi$$

$$0 \in [-1,57; 4,81]$$



Croissance



$$f'(x) = -3 \sin x$$

$$-3 \sin\left(-\frac{\pi}{2}\right) = 3$$

$$-3 \sin\left(\frac{\pi}{2}\right) = -3$$

$$-3 \sin\left(\frac{3\pi}{2}\right) = 3$$

$$f(0) = 3 \cos(0) - 2 = 3 - 2 = 1$$

$$f(\pi) = 3 \cos(\pi) - 2 = -3 - 2 = -5$$

③ graphe

