$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}\quad \Bigg\}\quad \varphi = \frac{1+\sqrt{5}}{2}$$

$$\varphi = 1+\frac{1}{\varphi}$$

$$:\varphi \qquad \cdot\varphi$$

$$\varphi\cdot\varphi = \varphi\cdot 1+\frac{1}{\varphi}\cdot\varphi$$

$$\boxed{\varphi^2 = \varphi + 1}$$

$$\boxed{\varphi^2 = \varphi + 1}$$

$$\varphi+1 = \frac{1+\sqrt{5}}{2}+1 = \frac{1+\sqrt{5}}{2}+\frac{2}{2} = \frac{3+\sqrt{5}}{2}$$

$$= \frac{3}{2}+\frac{1}{2}\sqrt{5}$$

$$\left(\sqrt{5}\right)^2 = 5$$

$$\varphi^2 = \frac{\left(1+\sqrt{5}\right)^2}{4} = \frac{1+2\sqrt{5}+5}{4}$$

$$= \frac{6+2\sqrt{5}}{4} = \frac{\cancel{2}\left(3+\sqrt{5}\right)}{\cancel{2}\cdot 2} = \frac{3+\sqrt{5}}{2}$$

$$\Rightarrow \varphi^2 = \varphi+1$$

$$\varphi = 1 + \frac{1}{\varphi}$$

conjugué

$$\frac{1}{\varphi} = \frac{2}{1+\sqrt{5}} = \frac{2}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}}$$

$$(A+B) \cdot (A-B) = A^2 - B^2$$

$$= \frac{2(1-\sqrt{5})}{1^2-(\sqrt{5})^2} = \frac{2-2\sqrt{5}}{1-5}$$

$$= \frac{2-2\sqrt{5}}{-4} = \frac{-1+\sqrt{5}}{2}$$

$$1+\frac{1}{\varphi} = 1 + \frac{-1+\sqrt{5}}{2} = \frac{2-1+\sqrt{5}}{2} = \frac{1+\sqrt{5}}{2} = \varphi$$

<u>Proposition</u> On pose $\varphi = \frac{1+\sqrt{5}}{2}$

Alors $\varphi^2 = \varphi + 1$

<u>preuve par calcul:</u>

$$\left.\begin{aligned} \varphi+1 &= \frac{3+\sqrt{5}}{2} \\ \varphi^2 &= \frac{3+\sqrt{5}}{2} \end{aligned}\right\} \Rightarrow \varphi^2 = \varphi+1$$

Quelques calculs (voir ci-dessus)

CQFD

$\boxed{2.5.12 \quad j)}$

$$\frac{x+3}{3x-1} + \frac{1}{4} = \frac{2x-9}{4-12x} + 1$$

$$\frac{x+3}{3x-1} + \frac{1}{4} = \frac{2x-9}{4(1-3x)} + 1$$

$$4\cdot(-1)(3x-1) = -4(3x-1)$$

$$\frac{x+3}{3x-1} + \frac{1}{4} = -\frac{2x-9}{4(3x-1)} + 1 \qquad \Big|\cdot 4(3x-1)$$

$$\frac{4\cdot\cancel{(3x-1)}\cdot(x+3)}{\cancel{(3x-1)}} + \frac{\cancel{4}(3x-1)}{\cancel{4}} = \frac{-(2x-9)\cdot\cancel{4\cdot(3x-1)}}{\cancel{4\cdot(3x-1)}} + 4(3x-1)$$

$$4(x+3) + 3x-1 = -(2x-9) + 4(3x-1)$$

$$\cancel{4x}+12 \cancel{+3x}-1 = \cancel{-2x}+9 + \cancel{12x} - 4$$

$$12-1-9+4 = -4x-3x-2x+12x$$

$$6 = 3x$$

$$\boxed{x=2}$$

2.4.4 g)

$$\frac{x+2}{(x+5)(x+2)} - \frac{x-3}{(x-3)(x-5)} + \frac{x^2-15}{(x-5)(x+5)} = \frac{?}{(x+2)(x-3)(x-5)(x+5)}$$

$$(x+2)(x-3)(x-5) - (x-3)(x+5)(x+2) + (x^2-15)(x-3)(x+2) =$$

$$(x+2)(x-3)(\underbrace{x-5-(x+5)}_{-10}) + (x^2-15)(x-3)(x+2) =$$

$$(x+2)(x-3)(-10+x^2-15) =$$

$$(x+2)(x-3)(x^2-25)$$

$$\frac{(x+2)(x-3)(x^2-25)}{(x+2)(x-3)(x+5)(x-5)} = 1$$

$$
\begin{cases}
x - y - z = 6 & L_1 \\
x - 2y - 3z = 10 & L_2 \\
5x + 6y + z = 2 & L_3
\end{cases}
$$

$L_2 \leftarrow L_2 - L_1$

$L_3 \leftarrow L_3 - 5L_1$

$$
\begin{cases}
x - y - z = 6 & L_1 \\
-y - 2z = 4 & L_2 \\
11y + 6z = -20 & L_3
\end{cases}
$$

$L_3 \leftarrow L_3 + 11L_2$

$$
\begin{cases}
x - y - z = 6 \\
y + 2z = -4 \\
-16z = 24
\end{cases}
\qquad
\begin{cases}
x = \frac{7}{2} \\
y = -1 \\
z = -\frac{3}{2}
\end{cases}
$$

$$\left(S = \left\{ \left(\frac{7}{2} ; -1 ; -\frac{3}{2} \right) \right\} \right)$$

$$\frac{3x+1}{8} - \frac{x^2+5}{4} = \frac{55}{2}$$

$$\frac{3x+1}{8} - \frac{2(x^2+5)}{8} = \frac{220}{8} \quad \cdot 8$$

$$\cancel{3x+1} - \cancel{2x^2} - 10 = 220$$

$$2x^2 - 3x + 229 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 8 \cdot 229}}{4} \in \mathbb{C}$$

car $9 - 8 \cdot 229 < 0 \Rightarrow$ Pas de solution dans $\mathbb{R}$