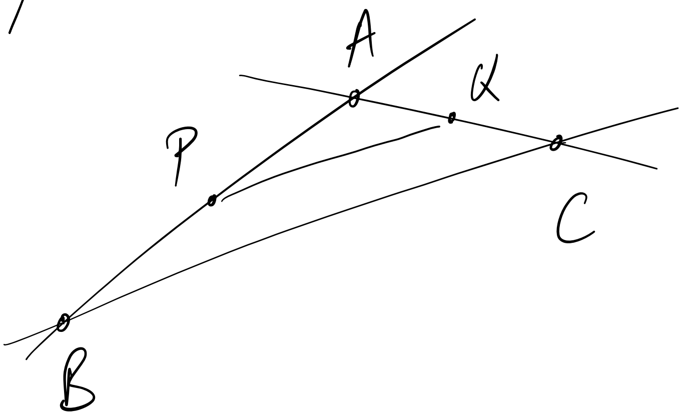
1.10

Démontrer le théorème du segment moyen

Géométrie plane

Prop. Soit ABC un triangle qcq.

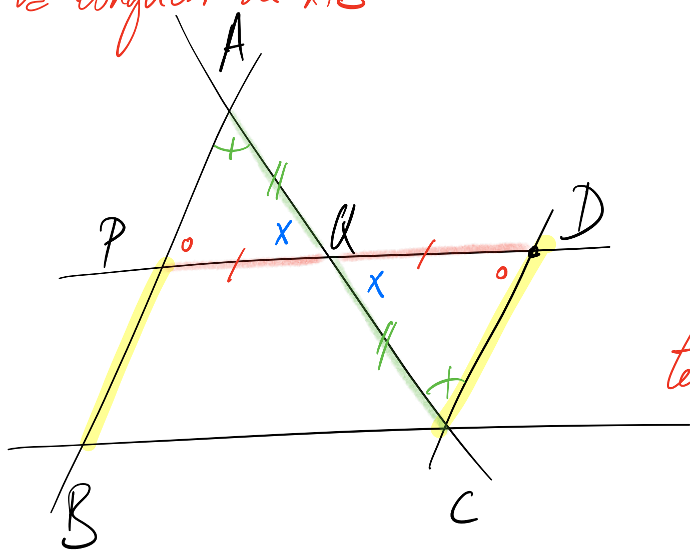


Soit P, Q les milieux de AB, AC, resp.

Alors $d_{PQ} \parallel d_{BC}$ et $\overline{PQ} = \frac{1}{2}\overline{BC}$.

Note : $\overline{AB}$ est la longueur de AB

Preuve.



Soit D sur d_{PQ} tq. $\overline{QD} = \overline{PQ}$

tel que

D'après un cas d'isométrie des $\triangle$ (2 côtés un angle entre les deux côtés) $\triangle PAQ \sim \triangle QDC$

$\Rightarrow \widehat{QPA} = \widehat{QDC} \Rightarrow d_{AB} \parallel d_{DC}$

$\Rightarrow DC \parallel PB$ et $\overline{DC} = \overline{AP} = \overline{PB}$

$\Rightarrow$ PDCB a une paire de côtés $\parallel$ et isométriques

$\Rightarrow$ PDCB est un $\parallel$ogramme.

$d_{PD} \parallel d_{BC}$ et $\overline{PD} = \overline{BC}$

$\Rightarrow d_{PQ} \parallel d_{BC}$ et $\overline{PQ} = \frac{1}{2}\overline{PD}$ par constr.

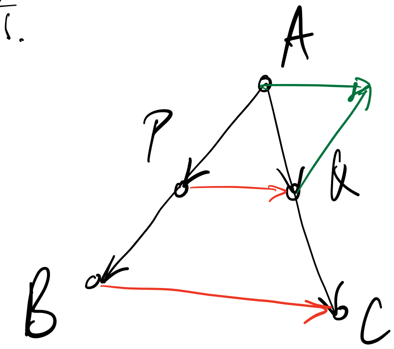
$$= \frac{1}{2}\overline{BC}$$

CQFD

Géométrie vectorielle

$P = A + \frac{1}{2}\vec{AB}$

$Q = A + \frac{1}{2}\vec{AC}$



Prop. Soit A, B, C trois points.

Soit P tq. $\vec{AP} = \frac{1}{2}\vec{AB}$

Q tq. $\vec{AQ} = \frac{1}{2}\vec{AC}$

Alors $\vec{PQ} = \frac{1}{2}\vec{BC}$

preuve:

$$\vec{AB} + \vec{BC} = \vec{AC}$$

Chasles

$$\vec{BC} = \vec{AC} - \vec{AB}$$

$$\vec{AP} = \frac{1}{2}\vec{AB} \quad / \quad \vec{AQ} = \frac{1}{2}\vec{AC}$$

$$\vec{BC} = 2\vec{AQ} - 2\vec{AP}$$

$$\vec{BC} = 2(\vec{AQ} - \vec{AP})$$

$$= 2(\vec{AQ} + \vec{PA})$$

$$= 2(\vec{PA} + \vec{AQ}) = 2\vec{PQ}$$

CQFD

1.1.4

b) $\cancel{\overrightarrow{BC}} + \overrightarrow{DE} + \cancel{\overrightarrow{DC}} + \cancel{\overrightarrow{AD}} + \cancel{\overrightarrow{EB}} =$

$\overrightarrow{EB} + \overrightarrow{BC} + \overrightarrow{AD} + \overrightarrow{DC} =$

$\overrightarrow{EC} + \overrightarrow{AC} + \overrightarrow{DE} =$

$\overrightarrow{DE} + \overrightarrow{EC} + \overrightarrow{AC} =$

$\boxed{\overrightarrow{DC} + \overrightarrow{AC}}$

$\overrightarrow{EE}$

$\circ\ E$

$$\overrightarrow{BD} + \overrightarrow{AB} + \overrightarrow{DC} =$$

$$\underbrace{\overrightarrow{AB} + \overrightarrow{BD}} + \overrightarrow{DC} =$$

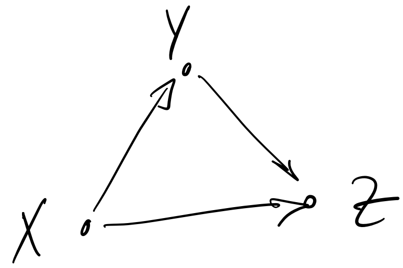
$$\overrightarrow{AD} + \overrightarrow{DC} =$$

$$\overrightarrow{AC}$$

$$\boxed{\overrightarrow{XY} + \overrightarrow{YZ} = \overrightarrow{XZ}}$$

$$-\overrightarrow{XY} = \overrightarrow{YX}$$

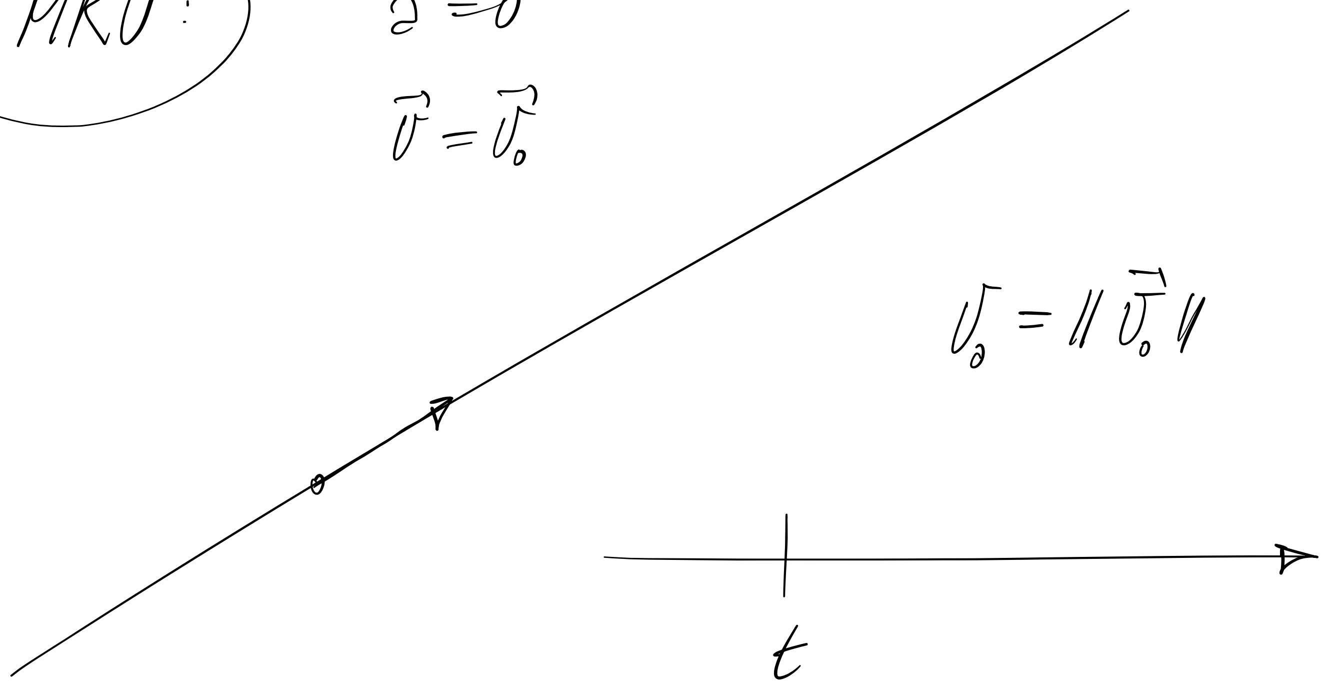
$$\overrightarrow{XX} = \vec{0}$$



MRU:

$\vec{a} = 0$

$\vec{v} = \vec{v}_0$



$v_0 = \|\vec{v}_0\|$

$$\boxed{x(t) = v_0 t}$$

$$\boxed{ax^2+bx+c=0} \qquad a,b,c \in \mathbb{R}$$

$$a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=0 \qquad A^2+2AB+B^2 \qquad A=x \quad B=\frac{b}{2a}$$

$$a\left(\boxed{x^2+2x\cdot\frac{b}{2a}+\frac{b^2}{4a^2}}-\frac{b^2}{4a^2}+\frac{c}{a}\right)=0 \qquad \left(\frac{b}{2a}\right)^2=\frac{b^2}{(2a)^2}=\frac{b^2}{4a^2}$$

$$a\left(\boxed{\left(x+\frac{b}{2a}\right)^2}-\left(\frac{b^2}{4a^2}-\frac{c}{a}\right)\right)=0$$

$$a\left(\underbrace{\left(x+\frac{b}{2a}\right)^2}_{x^2}-\underbrace{\left(\frac{b^2}{4a^2}-\frac{4ac}{4a^2}\right)}_{y^2}\right)=0$$

$$x^2-y^2=(x+y)(x-y)$$

$$a\left(x+\frac{b}{2a}+\sqrt{\frac{b^2-4ac}{4a^2}}\right)\left(x+\frac{b}{2a}-\sqrt{\frac{b^2-4ac}{4a^2}}\right)=0$$

$$\boxed{a\left(x-\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\right)\left(x-\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\right)=0}$$

$$\Leftrightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$$\Delta=b^2-4ac$$

$$x^2 + \frac{b}{a}x + \underbrace{\frac{b^2}{4a^2} - \frac{b^2}{4a^2}}_{0}$$

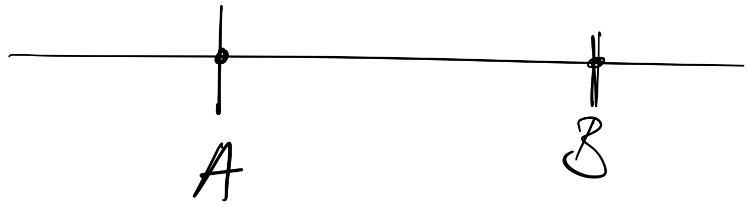
$$A^2 + 2AB + B^2$$

$$x^2 + 2x\frac{b}{2a}$$

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\overrightarrow{AP} = \frac{-5}{4}\overrightarrow{PB}$$



$$\overrightarrow{AP} = -\frac{5}{4}\left(\overrightarrow{PA} + \overrightarrow{AB}\right)$$

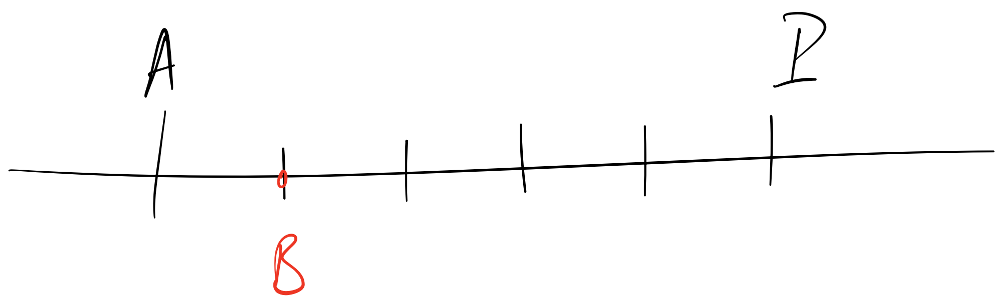
$$\overrightarrow{AP} = -\frac{5}{4}\overrightarrow{PA} - \frac{5}{4}\overrightarrow{AB}$$

$$\frac{5}{4}\overrightarrow{AB} = -\overrightarrow{AP} - \frac{5}{4}\overrightarrow{PA}$$

$$= -\overrightarrow{AP} + \frac{5}{4}\overrightarrow{AP}$$

$$\frac{5}{4}\overrightarrow{AB} = \frac{1}{4}\overrightarrow{AP} \quad \cdot \frac{4}{5}$$

$$\boxed{\overrightarrow{AB} = \frac{1}{5}\overrightarrow{AP}}$$



$$a \cdot \left(x^2 + \frac{b}{a} \cdot x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} \right) = ax^2 + bx + c$$

$$x^2 + \frac{2}{2} \cdot \frac{b}{a} \cdot x$$

$$x^2 + \frac{2}{2} \cdot x \cdot \frac{b}{a}$$

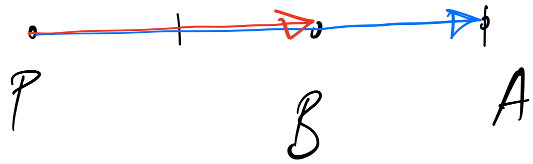
$$x^2 + \frac{2}{1} \cdot \frac{1}{2} \cdot x \cdot \frac{b}{a}$$

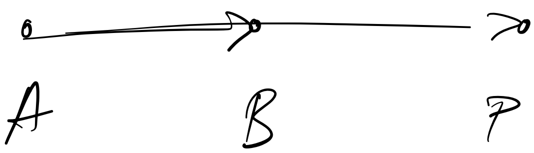
$$x^2 + \frac{2}{1} \cdot x \cdot \left(\frac{1}{2} \cdot \frac{b}{a} \right)$$

$$x^2 + 2x \cdot \frac{b}{2a}$$

$$\overrightarrow{PA} = -\frac{3}{2}\overrightarrow{BP}$$

$$= \frac{3}{2}\overrightarrow{PB} = 1{,}5\ \overrightarrow{PB}$$





$$\overrightarrow{AP} = 2\overrightarrow{AB}$$