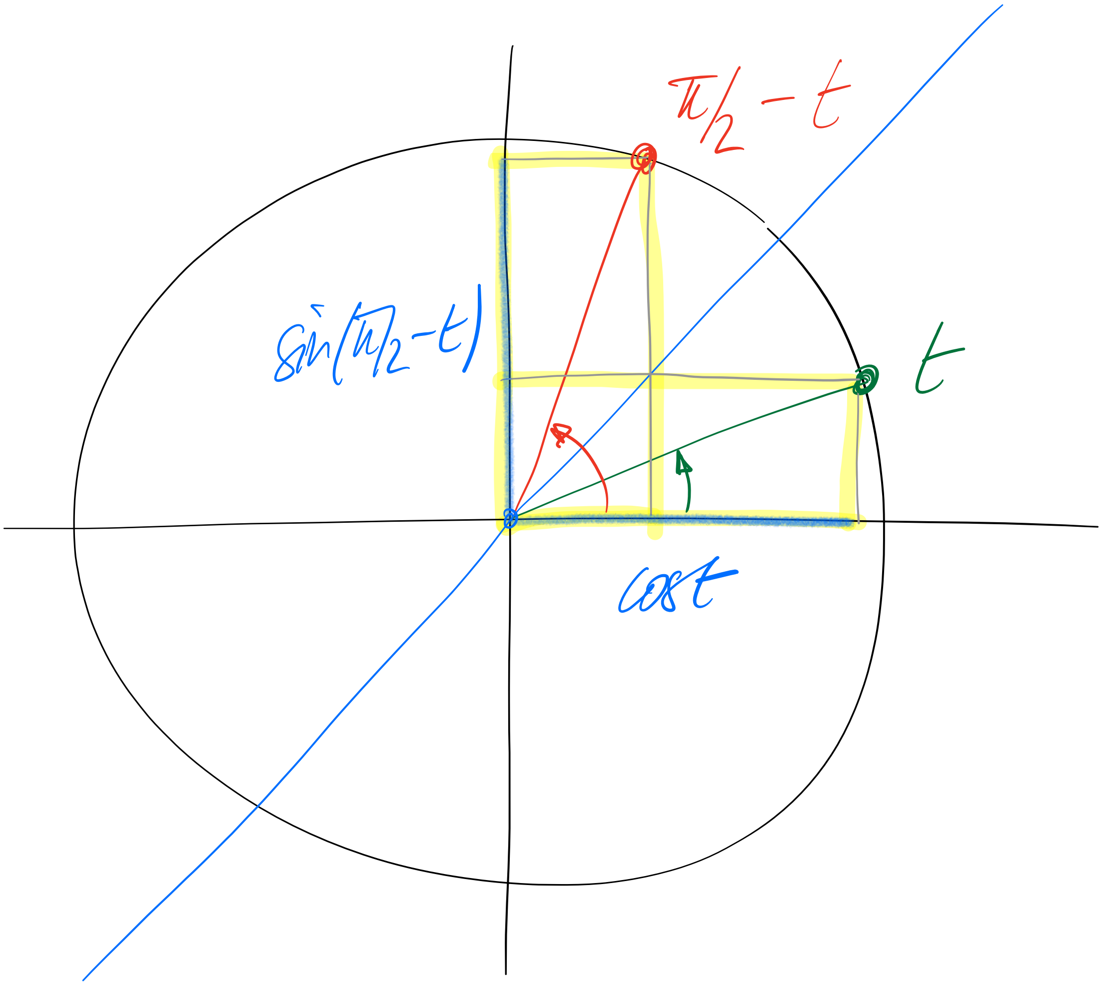
P1



Vu la symétrie d'axe tracé en bleu qui lie les deux points sur le cercle, on a bien :

$$\sin(\pi/2 - t) = \cos t$$

P2

$$\cos(x-y) = \cos(x+(-y))$$

$$= \cos x \cos(-y) - \sin x \sin(-y)$$

$$= \cos x \cos y + \sin x \sin y$$

On peut donc écrire :

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

La somme de ces deux lignes donne :

$$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$$

$$\Leftrightarrow \quad \cos x \cos y = \frac{1}{2}\left(\cos(x+y) + \cos(x-y) \right)$$

$\boxed{P3}$ a) $\sin\left(3t - \frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$

$$\Leftrightarrow \quad 3t - \frac{\pi}{2} = \arcsin\left(\frac{\sqrt{2}}{2}\right) + k \cdot 2\pi$$

$$3t - \frac{\pi}{2} = \pi - \arcsin\left(\frac{\sqrt{2}}{2}\right) + k \cdot 2\pi$$

$$\Leftrightarrow \quad 3t - \frac{\pi}{2} = \frac{\pi}{4} + k \cdot 2\pi$$

$$3t = \frac{\pi}{2} + \frac{\pi}{4} + k \cdot 2\pi$$

$$\boxed{t = \frac{\pi}{4} + k \cdot \frac{2\pi}{3}}$$

$$\Leftrightarrow \quad 3t - \frac{\pi}{2} = \pi - \frac{\pi}{4} + k \cdot 2\pi$$

$$\Leftrightarrow \quad 3t = \frac{\pi}{2} + \frac{3\pi}{4} + k \cdot 2\pi$$

$$3t = \frac{5\pi}{4} + k\cdot 2\pi$$

$$\boxed{t = \frac{5\pi}{12} + k\cdot\frac{2\pi}{3}}$$

b) $\sin x + \cos^2 x = 0$

$\sin x + 1 - \sin^2 x = 0$

$\sin^2 x - \sin x - 1 = 0$

$\sin x = t$

$t^2 - t - 1 = 0$

$t = \sin x$

$$t = \frac{1 \pm \sqrt{1+4}}{2} = \begin{cases} \cancel{\dfrac{\sqrt{5}+1}{2} \simeq 1{,}62 > 1} \\ \dfrac{1-\sqrt{5}}{2} \simeq -0{,}62 \end{cases}$$

On a donc :

$$\sin x = \frac{1-\sqrt{5}}{2}$$

$$x = \arcsin\left(\frac{1-\sqrt{5}}{2}\right) + k \cdot 2\pi$$

$$x = \pi - \arcsin\left(\frac{1-\sqrt{5}}{2}\right) + k \cdot 2\pi$$

$$\Leftrightarrow \boxed{\begin{aligned} x &\cong -0{,}666 + k\,2\pi \simeq 5{,}617 + k \cdot 2\pi \\ x &\cong 3{,}1416 + 0{,}666 + k\,2\pi \simeq 3{,}808 + k \cdot 2\pi \end{aligned}}$$