$\boxed{\text{Fonctions}}$

- fractions de polynômes
- trigo.
- pelo

Etude d'une fonction (exp.)

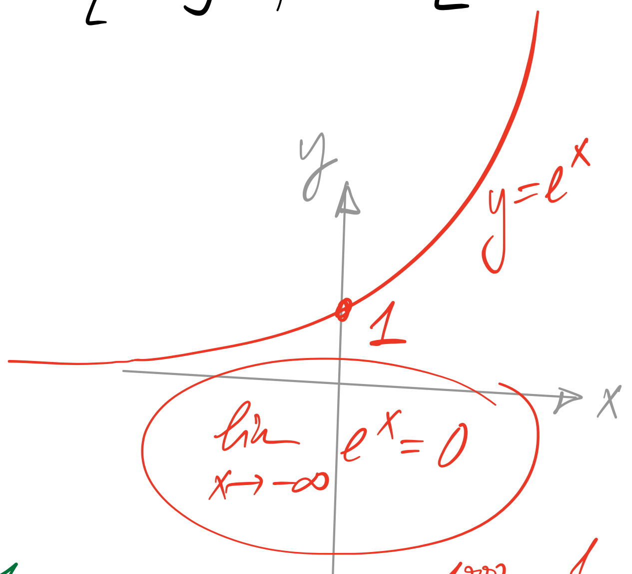
$$f(x) = \frac{e^x}{x-1}$$

a) D_f À exclure: $\frac{\ }{0}$ / $\sqrt{-}$ / $\log(\leq 0)$

$x - 1 = 0$ ⚠

$x = 1 \Rightarrow \boxed{D_f = \mathbb{R} - \{1\}} = \mathbb{R} \setminus \{1\}$

$=]-\infty; 1[\cup]1; +\infty[$



$\lim_{x \to -\infty} e^x = 0$

$e^{-1000} = \frac{1}{e^{1000}}$

__Zéros:__ $f(x) = 0$

$\Leftrightarrow \frac{e^x}{x-1} = 0$

$e^x = 0$ ⚠ $x \neq 1$

$x = \underbrace{\ln(0)}_{\text{impossible}} \Rightarrow \boxed{\text{Pas de zéro}}$

$\frac{e^x}{x-1}$

$\boxed{x = 10}$ $\frac{e^{10}}{9} > 0$

__Signe:__

x		1	
$f(x)$	$-$	$-\infty$ ‖ $+\infty$	$+$

$\boxed{x = -10}$ $\frac{e^{-10}}{-11} = \frac{-e^{-10}}{11}$

b) A.V.

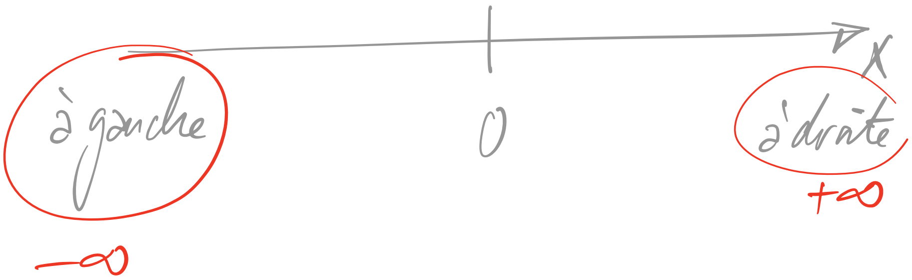
$$\lim_{x \to 1} f(x) = \lim_{x\to 1} \frac{e^x}{x-1} = « \frac{e^1}{1-1} » = « \frac{2{,}7}{0} » = \infty$$

$\neq 0$

à exclure

$$\boxed{\Rightarrow \text{A.V. en } \underbrace{x=1}_{\text{équation}}}$$

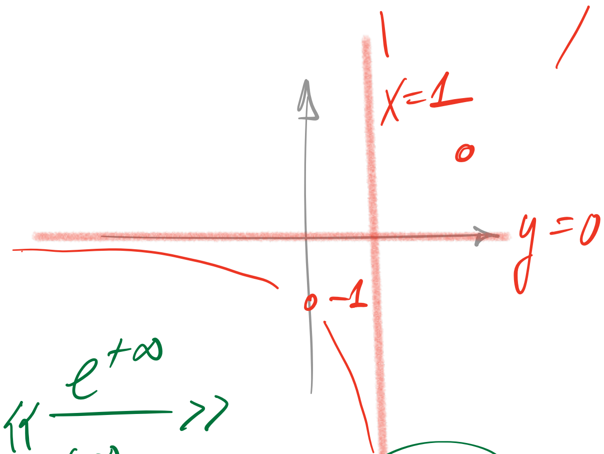
c) $\boxed{\text{A.H. à gauche}}$



$$\lim_{x\to -\infty} f(x) = \lim_{x\to-\infty} \frac{e^x}{x-1} = « \frac{e^{-\infty}}{-\infty} » = « \frac{0}{-\infty} » = 0$$

$\lim_{x\to-\infty} e^x$

$$\boxed{\text{A.H. en } y=0}$$

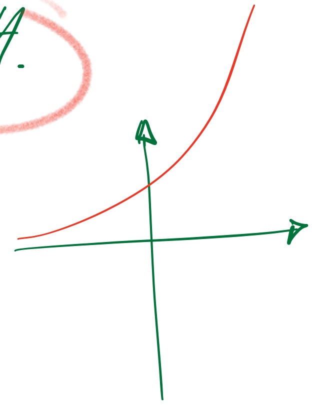


À droite ?

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{e^x}{x-1} = « \frac{e^{+\infty}}{+\infty} » = « \frac{\infty}{\infty} »$$

IND

Thm de Bernoulli – l'Hospital : B.–H.



$$\frac{e^x}{x-1} \xrightarrow{x\to+\infty} \text{« } \frac{+\infty}{+\infty} \text{ »}$$

Calculer $\boxed{\dfrac{(e^x)'}{(x-1)'}} = \dfrac{e^x}{1} = e^x \xrightarrow{x\to+\infty} +\infty$

$$\Rightarrow \lim_{x\to+\infty} \frac{e^x}{x-1} = +\infty \qquad \text{pas d'A.H. à droite}$$

Oblique ? (Pas d'oblique car e^x croît plus vite que n'importe quel polynôme)

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{e^x}{x-1}\cdot\frac{1}{x} = \lim_{x\to+\infty} \frac{e^x}{x^2-x}$$

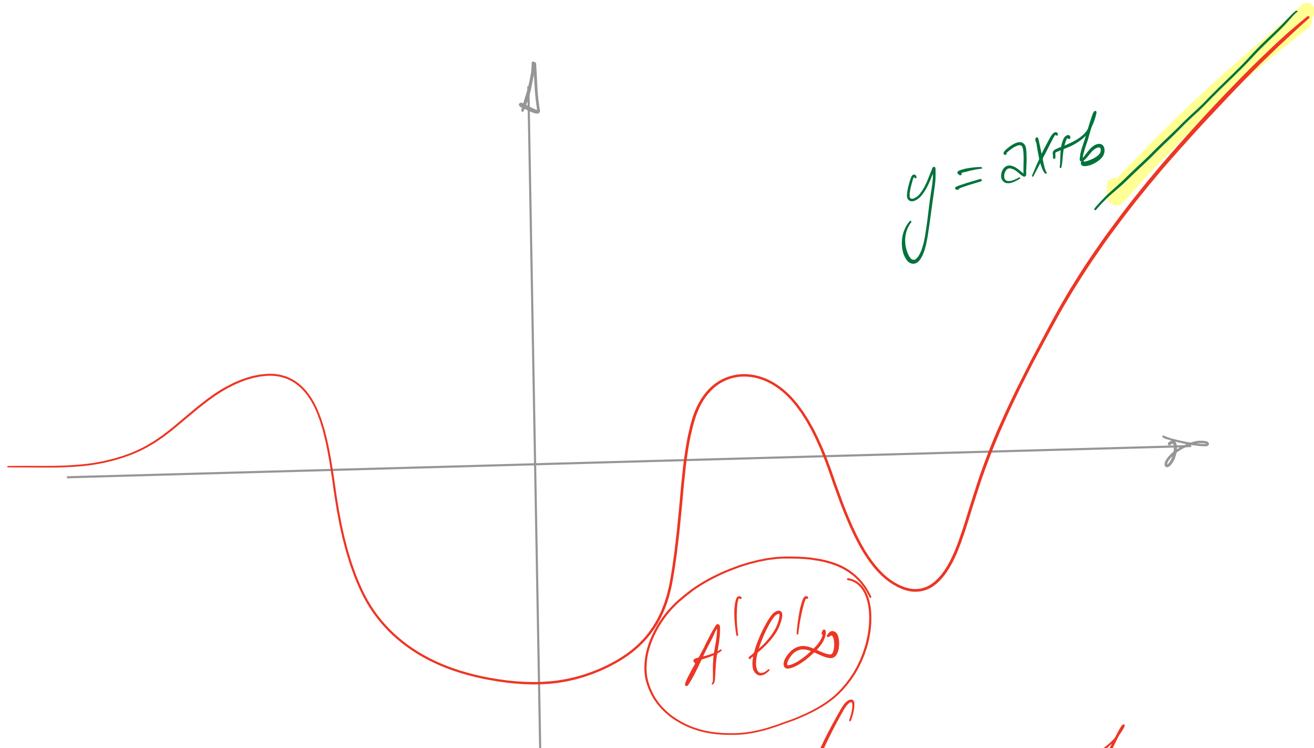
$$= \text{« } \frac{\infty}{\infty} \text{ » IND}$$

B.-H. (1)

$$\frac{(e^x)'}{(x^2-x)'} = \frac{e^x}{2x-1} \longrightarrow \text{« } \frac{\infty}{\infty} \text{ »}$$

B.-H. (2)

$$\frac{(e^x)'}{(2x-1)'} = \frac{e^x}{2} \longrightarrow \infty \quad \Rightarrow \text{Pas d'A.O.}$$



$$f(x) \sim ax+b$$

$$\frac{f(x)}{x} \sim a + \frac{b}{x}$$

$$\frac{f(x)}{x} \sim a$$

d) $$f'(x) = \left(\frac{e^x}{x-1}\right)'$$

$$\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}}$$

$$= \frac{(e^x)'(x-1) - e^x(x-1)'}{(x-1)^2}$$

$$= \frac{e^x(x-1) - e^x \cdot 1}{(x-1)^2} = \frac{e^x(x-1-1)}{(x-1)^2}$$

$$\Rightarrow f'(x) = \frac{e^x(x-2)}{(x-1)^2}$$