Réciproque d'une fonction

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto f(x)$$
$$f(y) \longleftarrow y$$

Exemple :

$$f(x) = \frac{1-x}{2x+3}$$ homographique

$f(x)$? $\quad y \longmapsto$?

$$y = \frac{1-x}{2x+3}$$

$$\Leftrightarrow \quad 2xy + 3y = 1 - x$$

$$\Leftrightarrow \quad 2xy + x = 1 - 3y$$

$$\Leftrightarrow \quad x(2y+1) = 1 - 3y$$

$$\Leftrightarrow \quad x = \frac{1-3y}{2y+1}$$

$$f^{r}(y) = \frac{1-3y}{2y+1} \qquad f^{r}(x) = \frac{1-3x}{2x+1}$$

$$f(f^{r}(x)) \stackrel{?}{=} x \qquad f \circ f^{r} = id$$

$$f^{r}(f(x)) \stackrel{?}{=} x \qquad f^{r} \circ f = id$$

$$id(x) = x \quad \forall x$$

$$f(x) = \frac{1-x}{2x+3} \qquad {}^{r}f(x) = \frac{1-3x}{2x+1}$$

$$f({}^{r}f(x)) = \frac{1 - \frac{1-3x}{2x+1}}{2 \cdot \frac{1-3x}{2x+1} + 3}$$

$$= \frac{2x+1-(1-3x)}{2x+1} \cdot \frac{2x+1}{2-6x+6x+3}$$

$$= \frac{5x \cdot \cancel{(2x+1)}}{\cancel{(2x+1)} \cdot 5} = \frac{5x}{5} = x$$

$${}^{r}f(f(x)) = \frac{1 - 3 \cdot \frac{1-x}{2x+3}}{2 \cdot \frac{1-x}{2x+3} + 1} = x$$

$$f(x) = \frac{1-x}{2x+3}$$

$2x+3 \neq 0$

$2x \neq -3 \;/\; x \neq -\frac{3}{2}$

$$\mathbb{R} - \left\{-\frac{3}{2}\right\} \xrightarrow{f} \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$\mathbb{R} - \left\{-\frac{3}{2}\right\} \xleftarrow{f^{-1}} \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$f^{-1}(x) = \frac{1-3x}{2x+1}$$

D_f

$D_{f^{-1}} = \operatorname{Im} f$

Sint $f, g : \mathbb{R} \to \mathbb{R}$

Def: $fog(x) = f(g(x))$

$\mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$

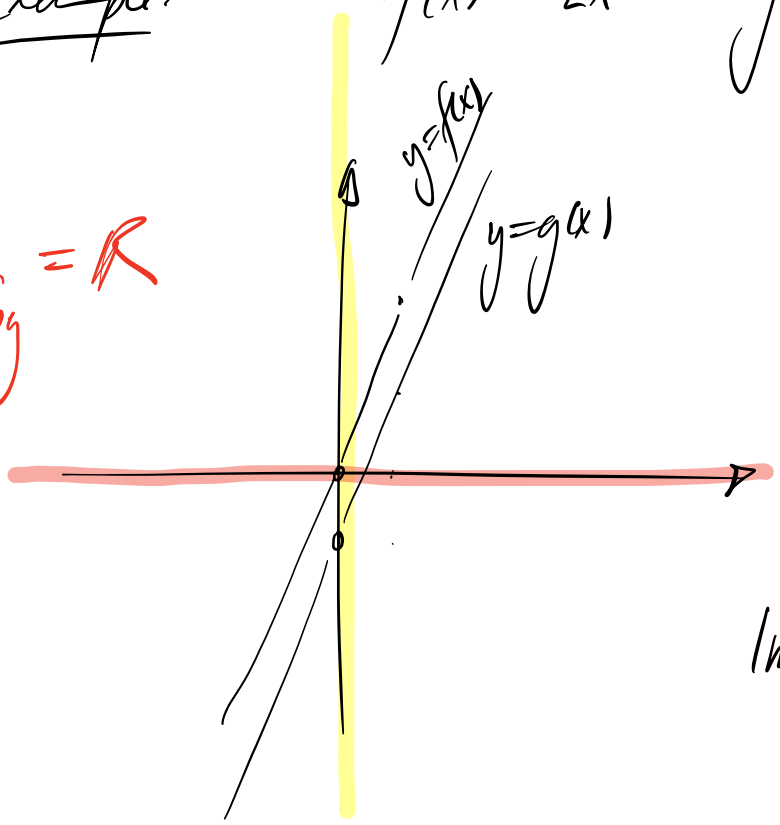
$x \longmapsto g(x)$

$x \in D_{fog} \quad : \quad x \in D_g$

$g(x) \in D_f \qquad x \in Im(g) \cap D_f$

Exemple: $f(x) = 2x \qquad g(x) = 2x - 1$

$D_{fog} = \mathbb{R}$

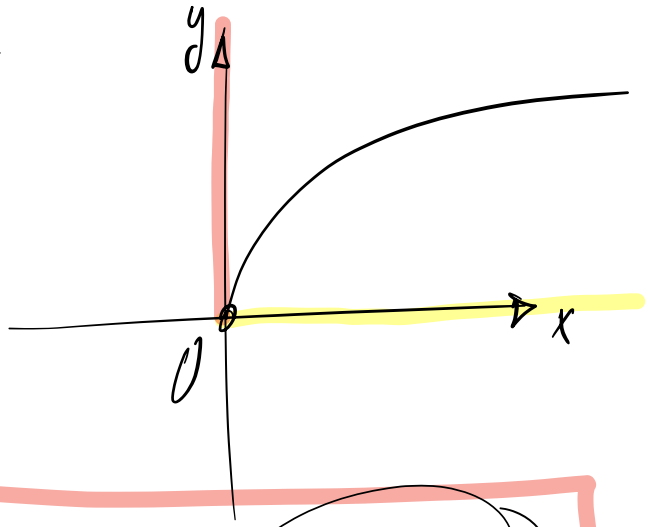


$fog(x) = 4x - 2$

$= 2 \cdot (\underbrace{2x-1}_{g(x)})$

$Im(g) = \mathbb{R} \qquad D_f = \mathbb{R}$

$f(x) = \frac{1}{x}$ $\quad g(x) = \sqrt{x}$



$f \circ g(x) = \frac{1}{\sqrt{x}}$

$D_g = [0; +\infty[$

$Im(g) = [0; +\infty[$

$D_f = \mathbb{R}^* = \mathbb{R} - \{0\}$ $\quad \Leftarrow \quad g(x) \neq 0$

$\Leftrightarrow \sqrt{x} \neq 0$

$\Leftrightarrow x \neq 0$

$(\mathbb{R} - \{0\}) \cap [0; +\infty[$

$=]0; +\infty[$

Soit $f, g : \mathbb{R} \to \mathbb{R}$ deux fonctions paires.

On a

$$(*) \begin{cases} f(x) = f(-x) \\ g(x) = g(-x) \end{cases} \quad \forall x \in D_f \cap D_g$$

Que dire de $f+g$?

Déf. $(f+g)(x) = f(x) + g(x)$

$$(f+g)(x) \overset{\text{déf}}{=} f(x) + g(x)$$

$$\overset{(*)}{=} f(-x) + g(-x) \overset{\text{déf}}{=} (f+g)(-x)$$

$\Rightarrow f+g$ est paire si f et g le sont.