

Exercice 1

Calculer l'angle entre les vecteurs

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

Cet angle est-il aigu ?

Exercice 2

Calculer l'angle entre les vecteurs

$$\vec{u} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$$

Cet angle est-il aigu ?

Exercice 3 Calculer tous les angles
du triangle ABC.

$$A(1; 2) \quad B(-1; 3) \quad C(-10; -5)$$

Exercice 4 Calculer tous les angles
du triangle CDE.

$$C = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad E = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Exo 1

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

$$\cos \varphi = \frac{2 \cdot (-4) + (-1) \cdot 1}{\sqrt{2^2 + 1^2} \cdot \sqrt{4^2 + 1^2}}$$

$$\cos \varphi = \frac{-8 - 1}{\sqrt{5} \cdot \sqrt{17}} = \frac{-9}{\sqrt{85}}$$

$$\simeq -0,97618706$$

$$\varphi \simeq \cos^{-1}(-0,97618706) \simeq 167,47^\circ$$

$$\simeq 167,5^\circ$$

L'angle est obtus ($> 90^\circ$).

$$\boxed{\text{Exo 2}} \quad \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \quad \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$$

$$\cos \varphi = \frac{-1 - 3 + 10}{\sqrt{1+9+25} \sqrt{1+1+4}}$$

$$\cos \varphi = \frac{6}{\sqrt{35} \sqrt{6}} = \frac{6}{\sqrt{210}}$$

$$\cos \varphi \simeq 0,41403934$$

$$\varphi \simeq \cos^{-1}(0,41403934) \simeq 65,54^\circ$$

$$\varphi \simeq 65,5^\circ$$

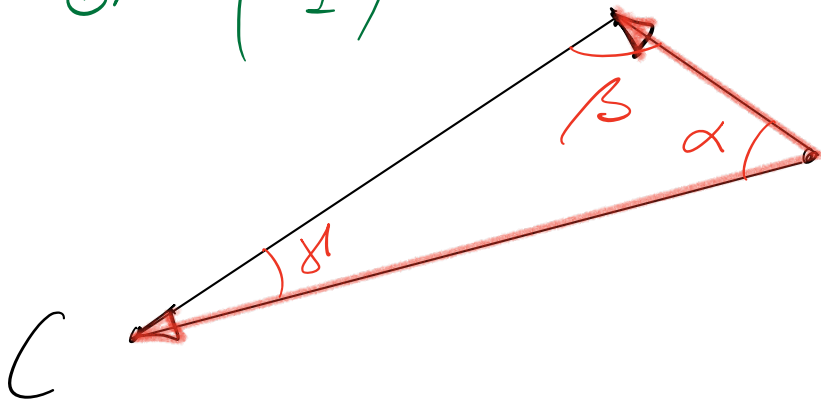
L'angle est aigu.

EX 3

$A(1; 2) \quad B(-1; 3) \quad C(-10; -5)$

$$\vec{BA} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \vec{AB} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -11 \\ -7 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} -9 \\ -8 \end{pmatrix}$$



$$\cos \alpha = \frac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \cdot \|\vec{AC}\|} = \frac{22 - 7}{\sqrt{5} \sqrt{121 + 49}} = \frac{15}{\sqrt{5} \sqrt{170}}$$

$$\cos \alpha \simeq 0,51449576$$

$$\alpha \simeq \cos^{-1}(0,51449576) \simeq 59,036$$

$$\simeq 59^\circ$$

$$\cos \beta = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \frac{-18 + 8}{\sqrt{5} \sqrt{81 + 64}} = \frac{-10}{\sqrt{5} \sqrt{145}}$$

$$\cos \beta \simeq 0,37139068$$

$$\beta \simeq \cos^{-1}(0,37139068)$$

$$\simeq 11,80^\circ \simeq 11,8^\circ$$

$$(\Rightarrow \gamma \simeq 180^\circ - 59^\circ - 11,8^\circ \simeq 9,2^\circ)$$

$$\cos \gamma = \frac{\vec{CB} \cdot \vec{CA}}{\|\vec{CB}\| \cdot \|\vec{CA}\|} = \frac{9 \cdot 11 + 7 \cdot 8}{\sqrt{145} \cdot \sqrt{170}}$$

$$= \frac{155}{\sqrt{145} \cdot \sqrt{170}} \simeq 0,98724112$$

$$\gamma \simeq \cos^{-1}(0,98724112) \simeq 9,16^\circ \simeq 9,2^\circ$$

Exo 4 $\vec{DC} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

$$\vec{DE} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

C

D

$$\vec{CD} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{CE} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$$

$$\vec{ED} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{EC} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

$$\cos \gamma = \frac{2}{1 \cdot \sqrt{8}} = \frac{2}{\sqrt{8}}$$

$$\simeq 0,707107$$

$$\Rightarrow \gamma \simeq \cos^{-1}(0,707107)$$

$$\gamma \simeq 45^\circ$$

$$\cos \delta = \frac{-1}{\sqrt{5}} \simeq -0,44721360$$

$$\delta \simeq \cos^{-1}(-0,44721360) \simeq 116,57^\circ$$

$$\simeq 116,6^\circ$$

$$\cos \varepsilon = \frac{6}{\sqrt{5}\sqrt{8}} = \frac{6}{\sqrt{40}} \simeq 0,94868330$$

$$\varepsilon \simeq \cos^{-1}(0,94868330) \simeq 18,43^\circ \simeq 18,4^\circ$$